Buckling & Crippling

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Rationale

Buckling/Crippling Modes Size a Large Fraction of Structure

Aircraft Structural Weight Breakdown by Failure Mode

Failure Mode	% Structural Weight	
	Airplane 1	Airplane 2
Tensile Strength	30.1	18.6
Compressive Strength	0.0	3.5
Crippling	14.3	19.5
Compression Surface Column Buckling	8.1	9.7
Shear or Compression Buckling	19.7	18.1
Aeroelastic Stiffness	14.1	11.6
Durability & Damage Tolerance	13.7	19.0
Total:	100.0	100.0

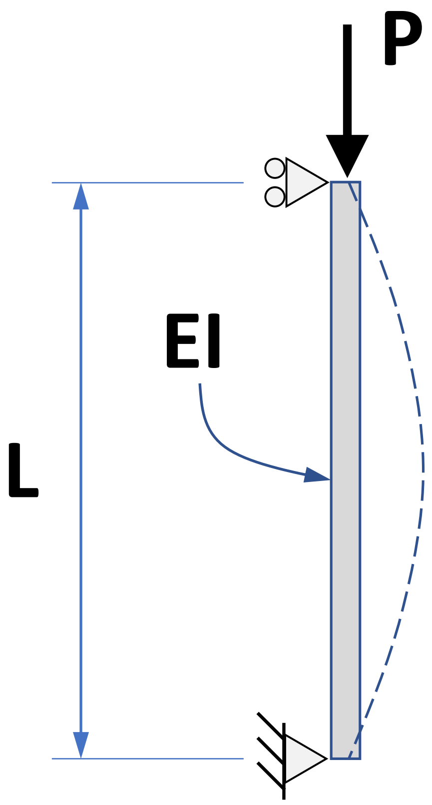
~ 40-50% Aircraft Structural Weight

From: Methodology for Evaluating Weight Savings from Basic Material Properties, Ekvall et al, 1982

Here we'll talk about column buckling and local buckling & crippling of thin-walled sections under compression

Column Buckling

Review Basic Column Buckling



In terms of Load →

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

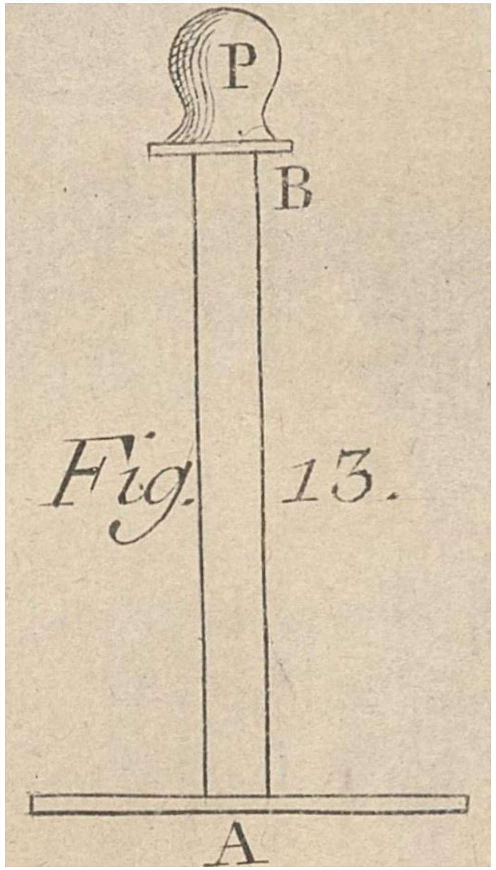
In terms of Stress →

$$\sigma_{cr} = \frac{\pi^2 E}{\left(L/\rho\right)^2}$$

ρ = radius of gyration (in)
L/ρ = slenderness ratio

$$\rho = \sqrt{\frac{I}{A}}$$

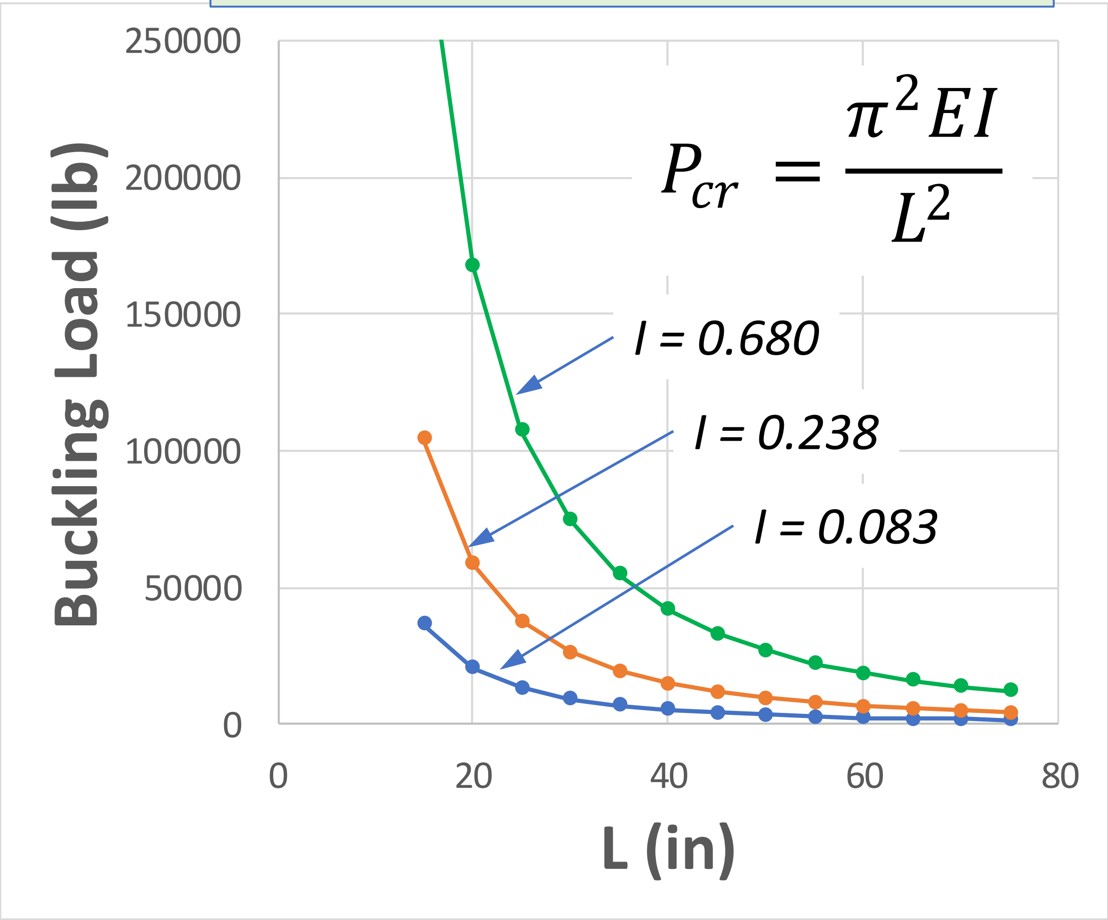
Euler, 1744



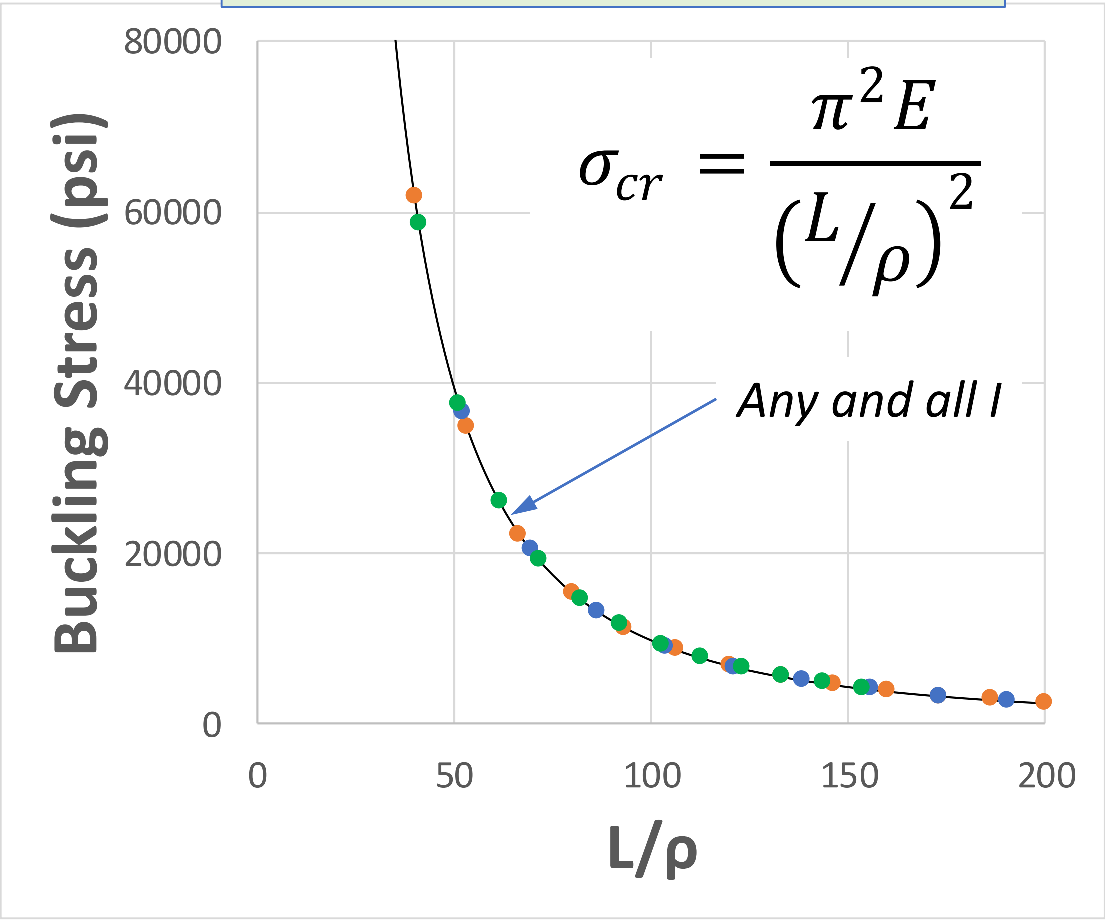
Note: σ_{cr} is an average stress, not a stress at a particular point

Advantage of using σ_{cr} rather than P_{cr}

Multiple curves for different Moments of Inertia

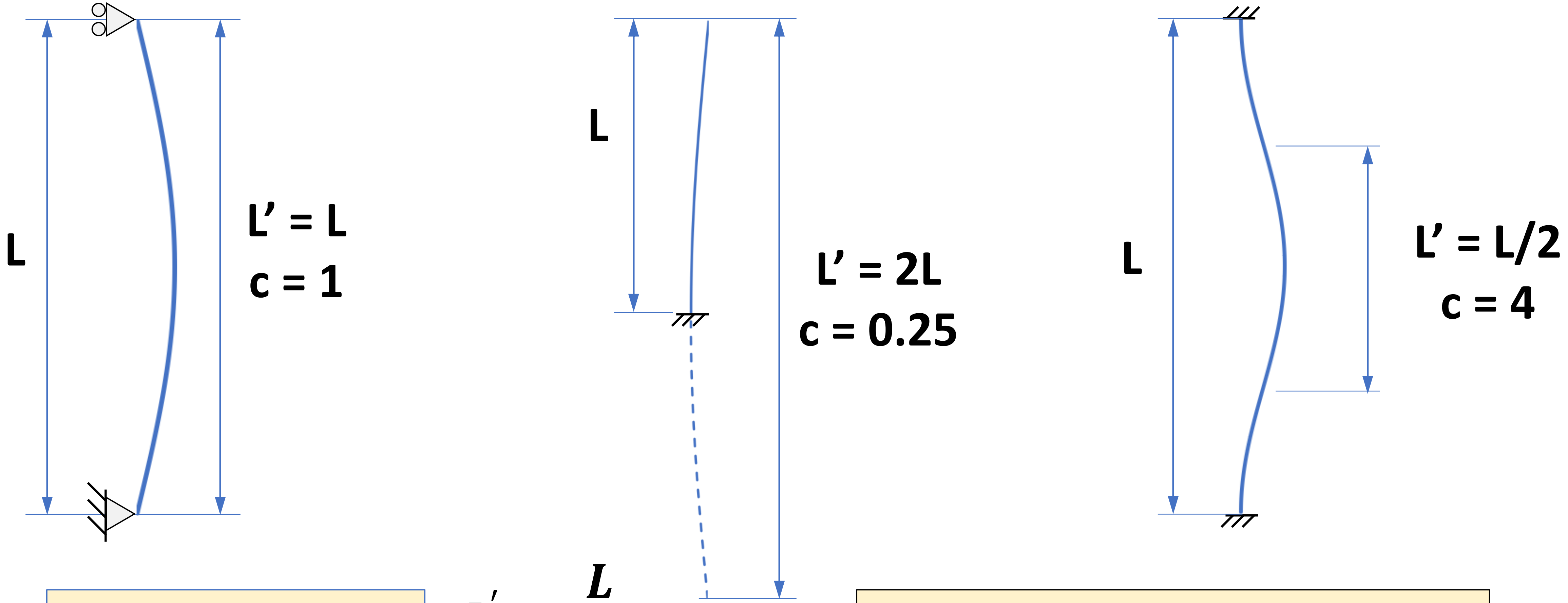


Single curve for all Moments of Inertia



Effective Length for Various Boundary Conditions

Pinned-Pinned | ***Fixed-Free*** | ***Fixed-Fixed***

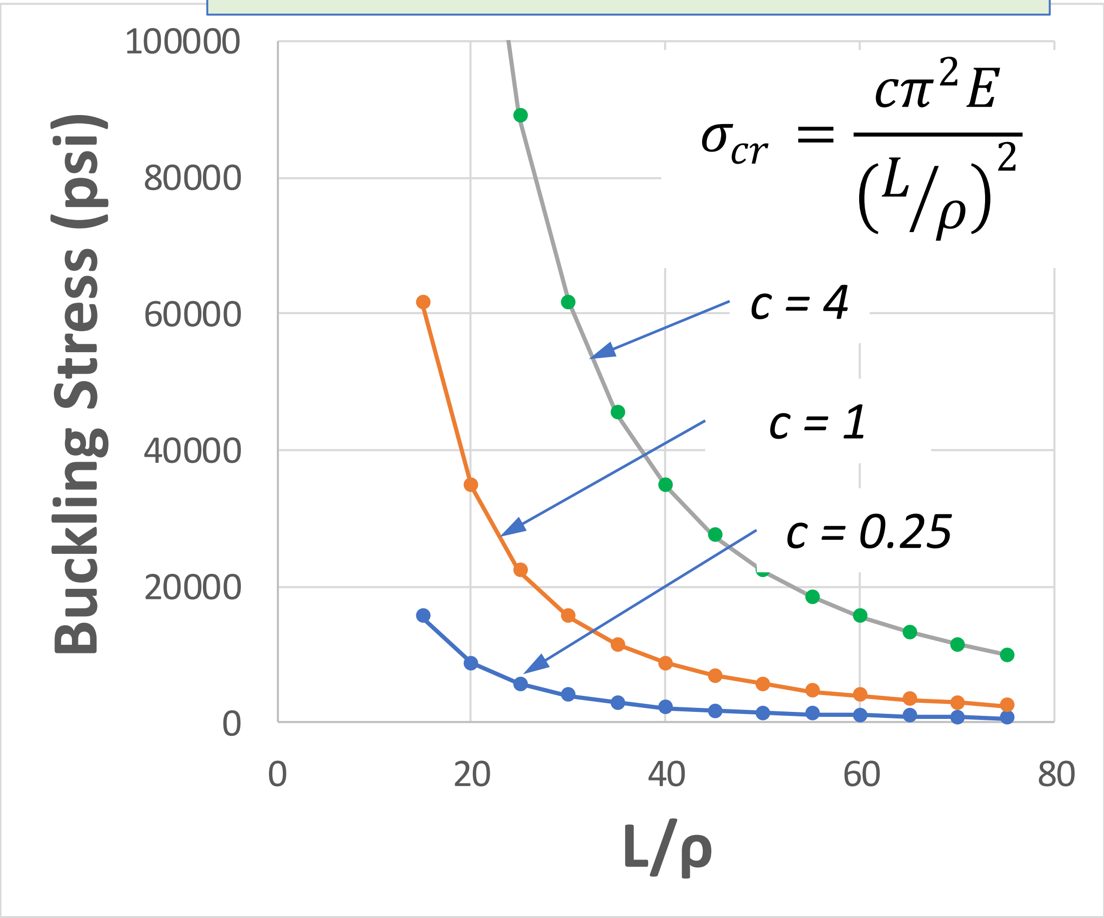


Effective Length → $L' = \dfrac{L}{\sqrt{c}}$

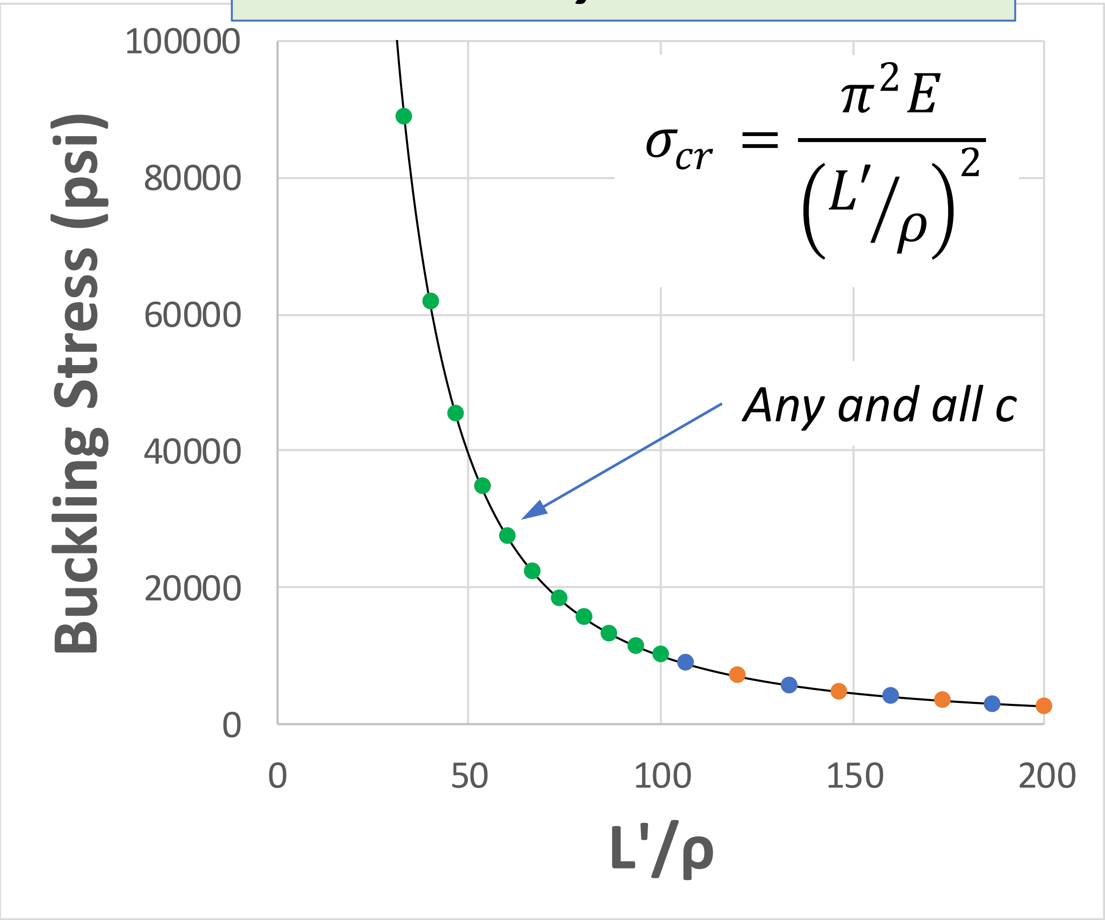
c = boundary condition coefficient

Advantage of using L' rather than L

Multiple curves for different Boundary Conditions

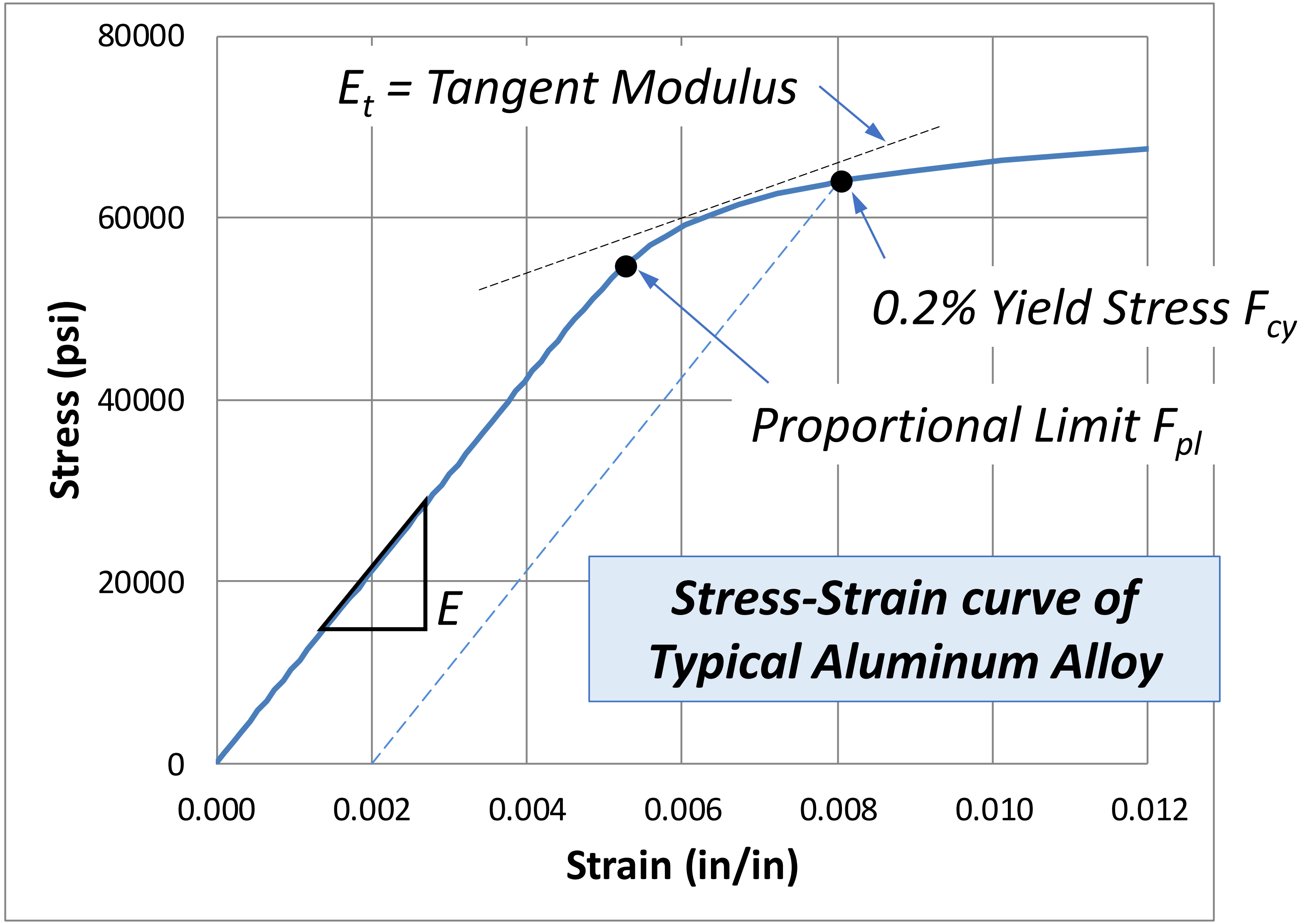


Single curve for all Boundary Conditions



Effect of Plasticity

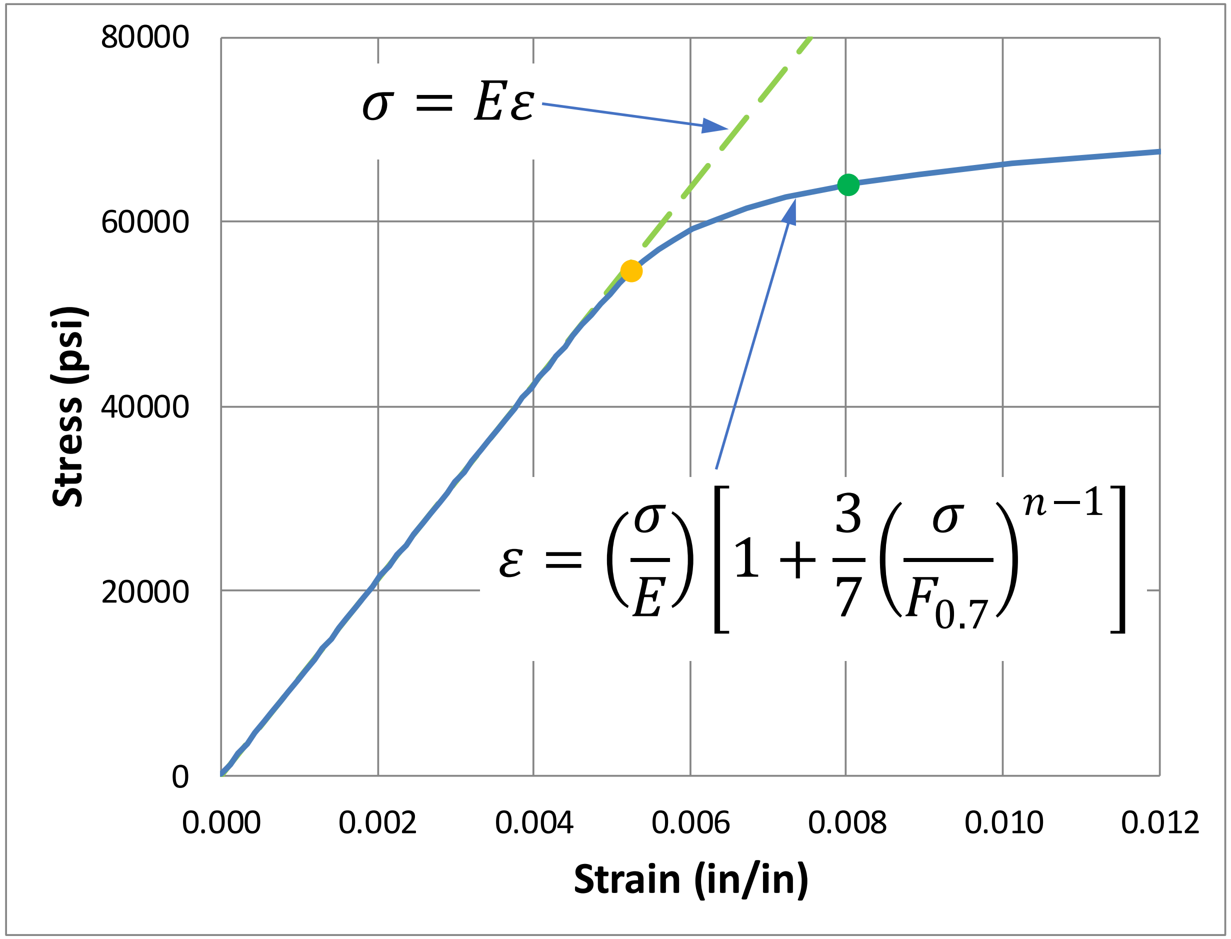
Onset of Plasticity reduces Modulus of Elasticity



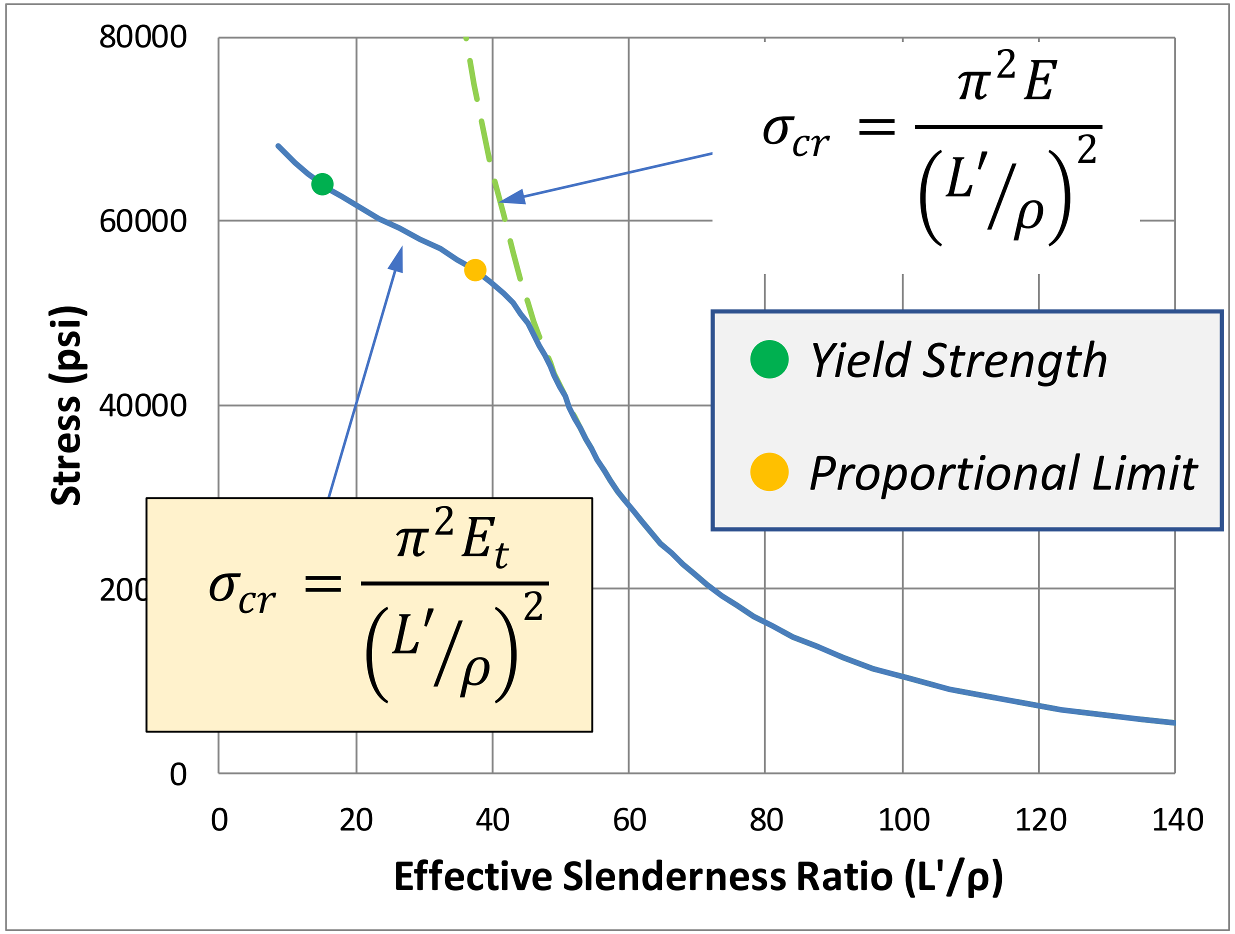
Note how the slope of the stress-strain curve (called the Tangent Modulus, E_t) decreases significantly BEFORE reaching the Yield Stress!

Buckling Loads are Lower in Plastic Range

Ramberg-Osgood can be used in Plastic Range



Use Tangent Modulus E_t in Buckling Formula



Buckling Equation with Plasticity Correction Factor

General formula for column buckling. Nonlinear because η is a function of σ. Can be solved by iteration or by the use of curves.

$$\sigma_{cr} = \frac{\eta \pi^2 E}{\left(L'/\rho\right)^2}$$

Plasticity Correction Factor →

$$\eta = \frac{E_t}{E}$$

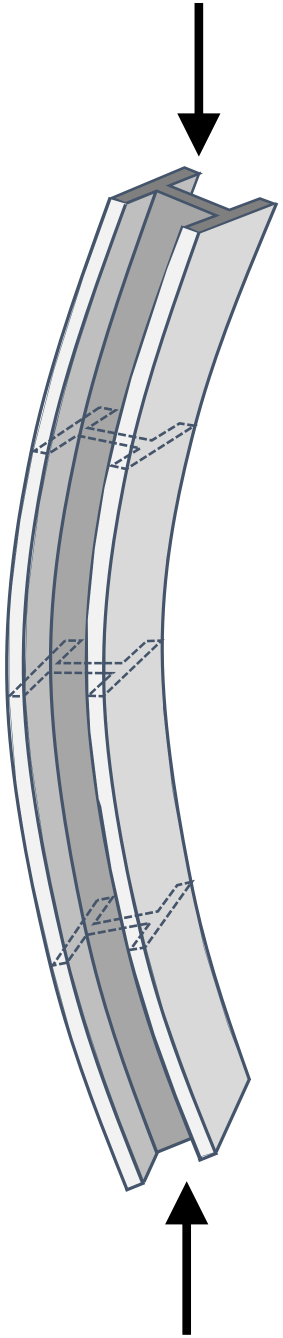
E = Young's Modulus (psi)
E_t = Tangent Modulus (psi)

Note: The Ramberg-Osgood representation of the stress-strain curve is often used in the aerospace industry to obtain the tangent modulus.

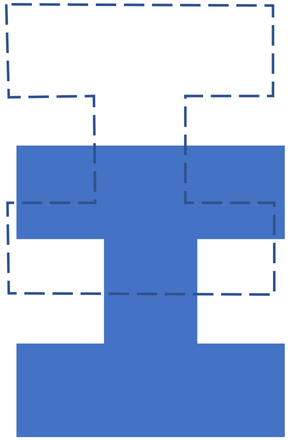
$$\varepsilon = \left(\frac{\sigma}{E}\right)\left[1 + \frac{3}{7}\left(\frac{\sigma}{F_{0.7}}\right)^{n-1}\right]$$

Local Buckling of Thin-Walled Sections

Global vs. Local Buckling



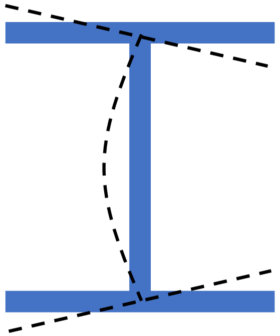
Global Buckling



<u>Compact Sections</u>

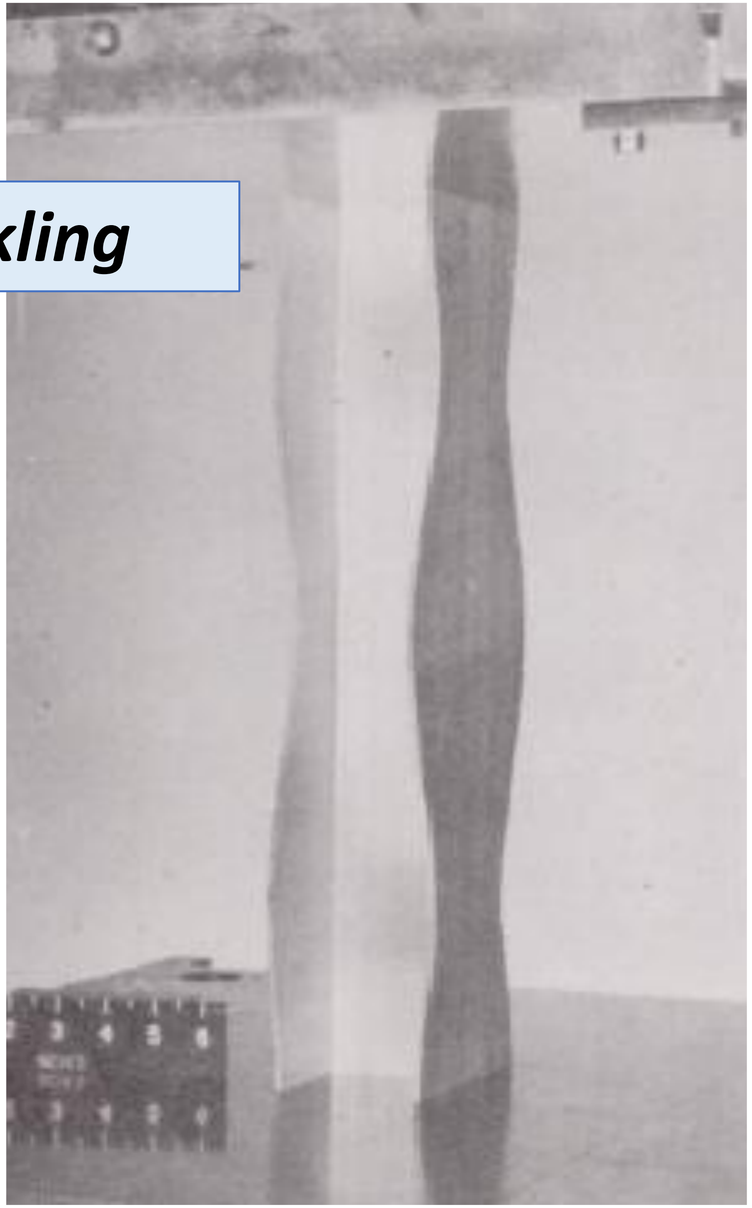
- ***Centerline bends***
- ***Cross-section translates undistorted***

Local Buckling



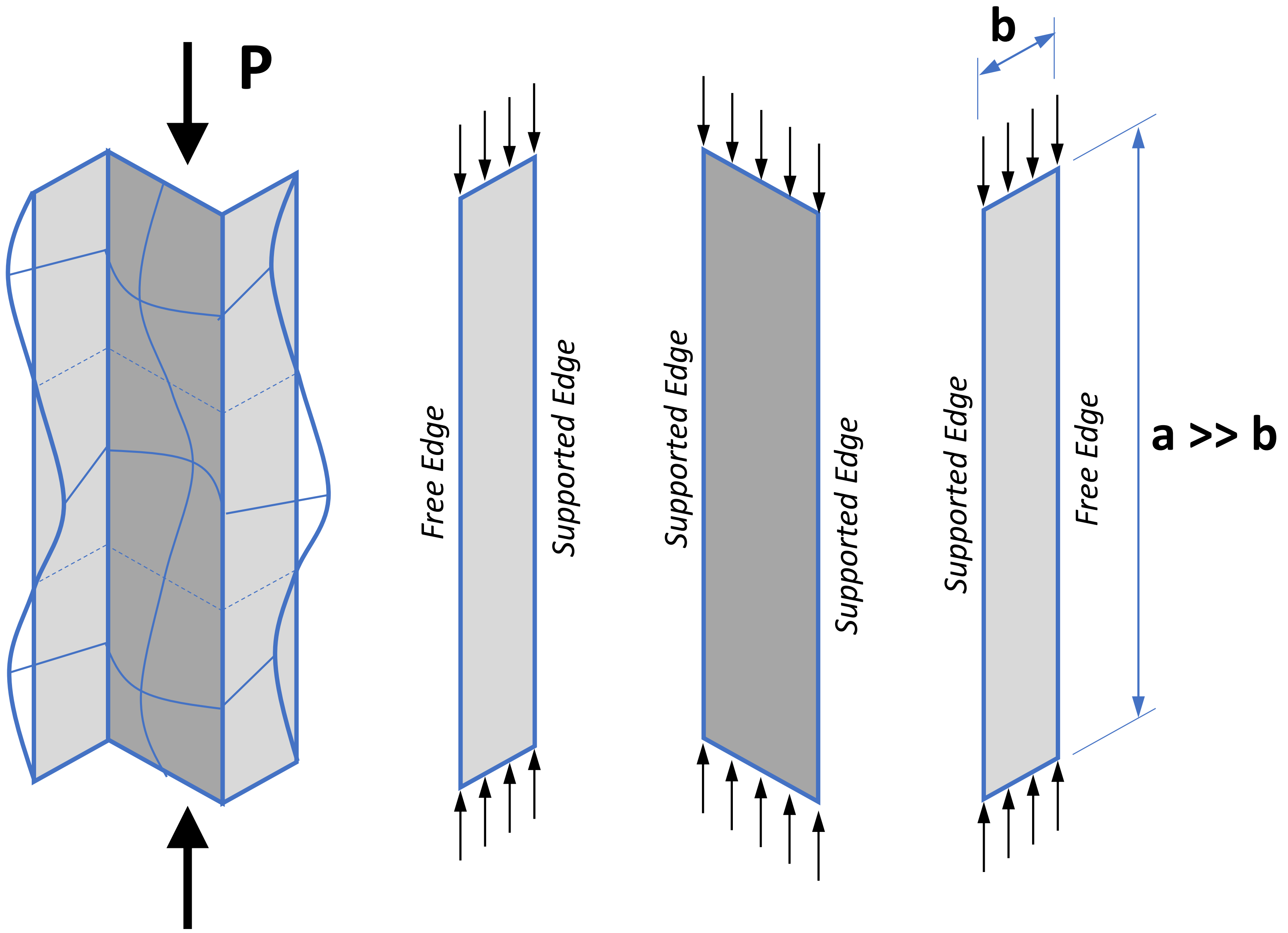
<u>Thin-Walled Sections</u>

- ***Corners remain straight***
- ***Cross-section distorts***



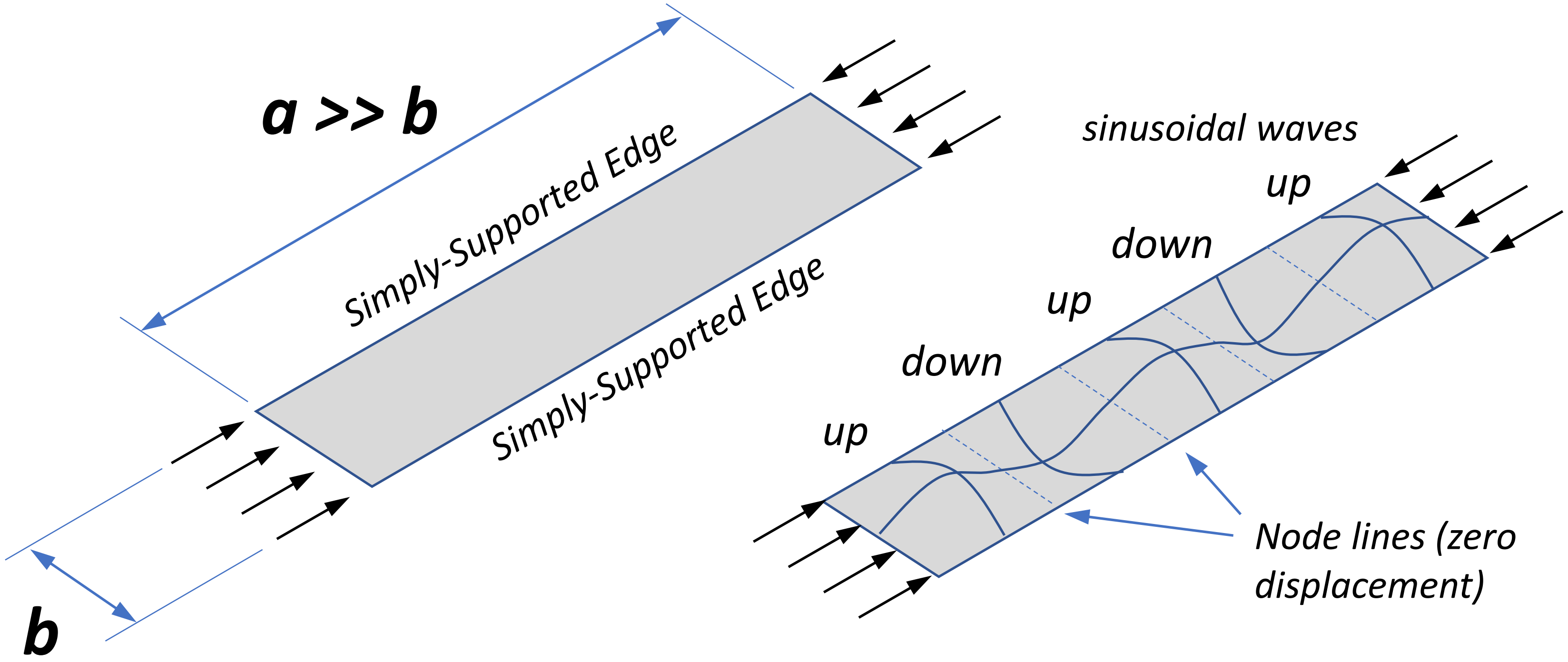
From: NACA-TN-1480

Idealization of Thin-Walled Sections



Thin-walled sections are often treated as an assembly of long flat plates, where long means a/b > ~ 3

Compression Buckling of a Long Flat Plate



A long plate with simply-supported edges buckles into nearly square waves

Plate Buckling Equation →

$$F_{cr} = \frac{k\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2$$

k = Buckling Coefficient which depends on Boundary Conditions on supported edges

Buckling Equations Compared

Column Buckling

$$F_{cr} = \frac{\eta \pi^2 E}{(L'/\rho)^2}$$

Column Buckling

- **_Driven by length L_**

Plate Buckling

$$F_{cr} = \frac{\eta k \pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2$$

η = Plasticity Correction Factor
(not same for plates & columns)

Modified buckling coefficient →

$$K = \frac{k\pi^2}{12(1-\nu^2)}$$

Effective b/t →

$$(b/t)_e = \frac{(b/t)}{\sqrt{K}}$$

Plate Buckling

- **_Driven by width b_**

Alternate Form →

$$F_{cr} = \frac{\eta E}{(b/t)_e^2}$$

Buckling Curves Compared

Column Buckling

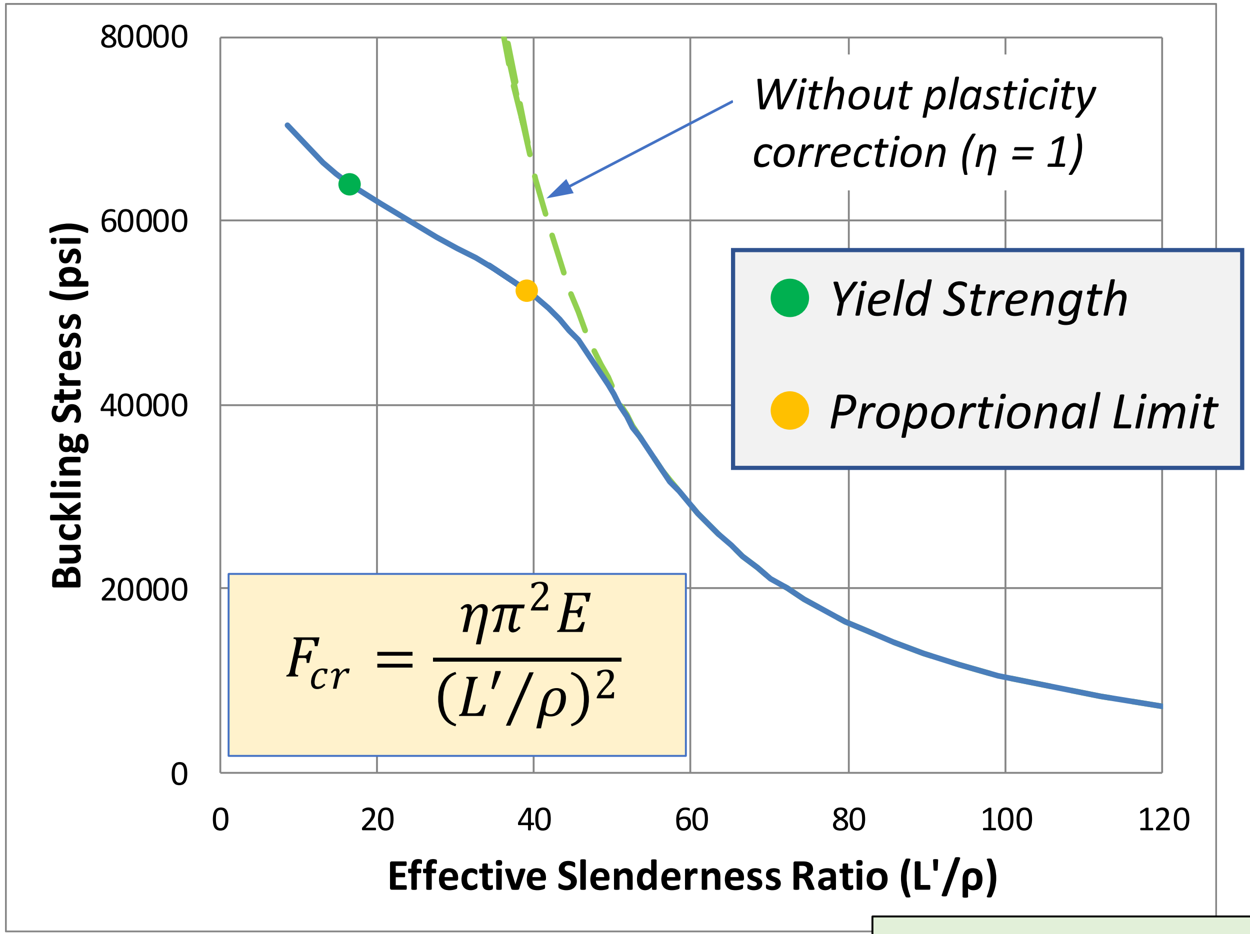
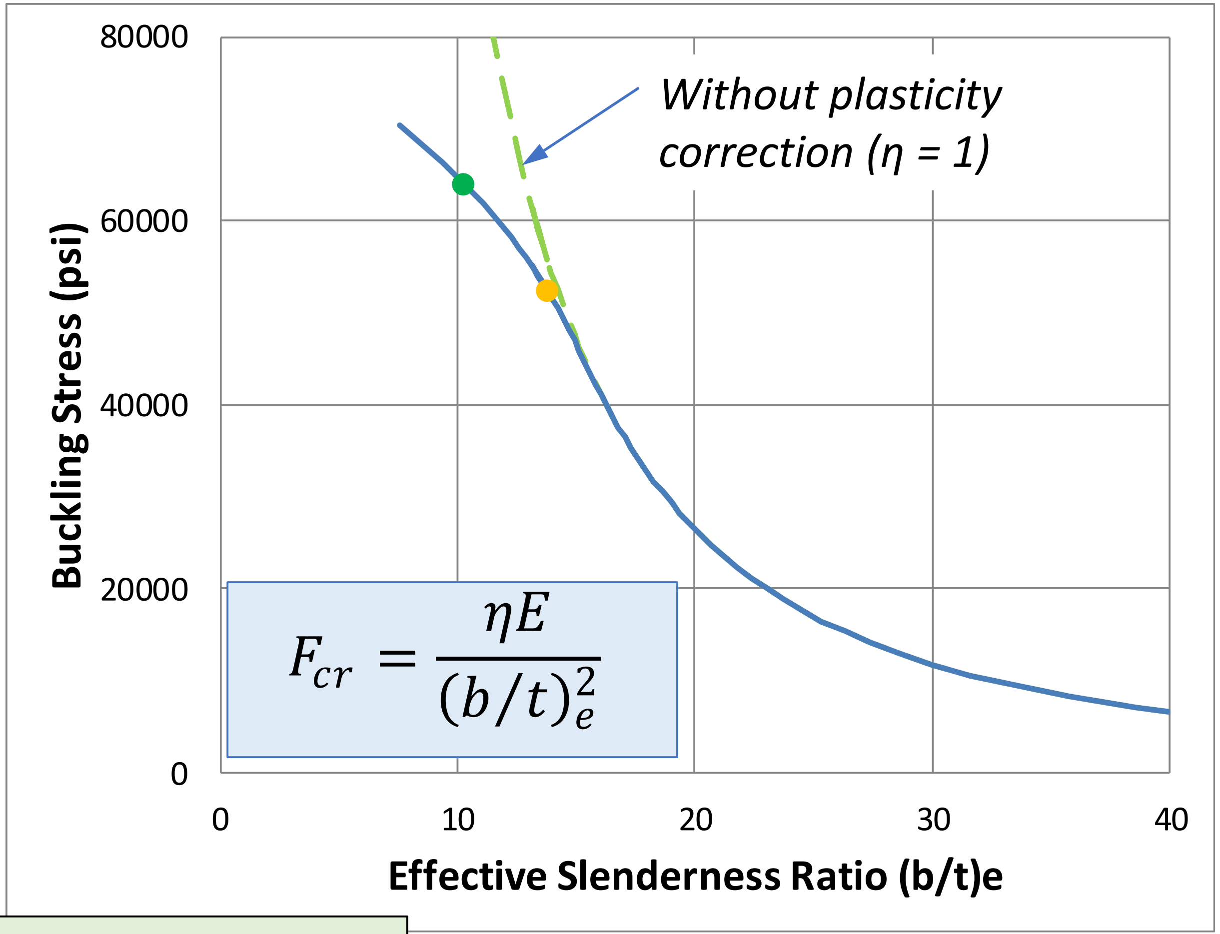
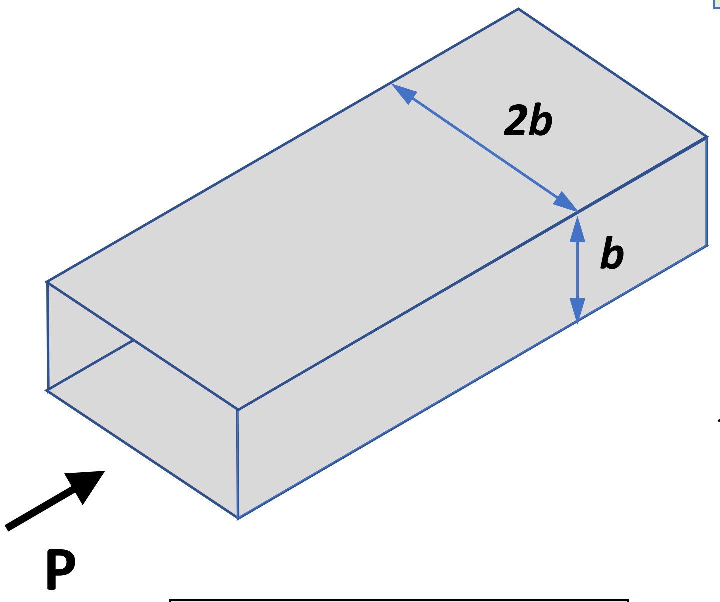


Plate Buckling



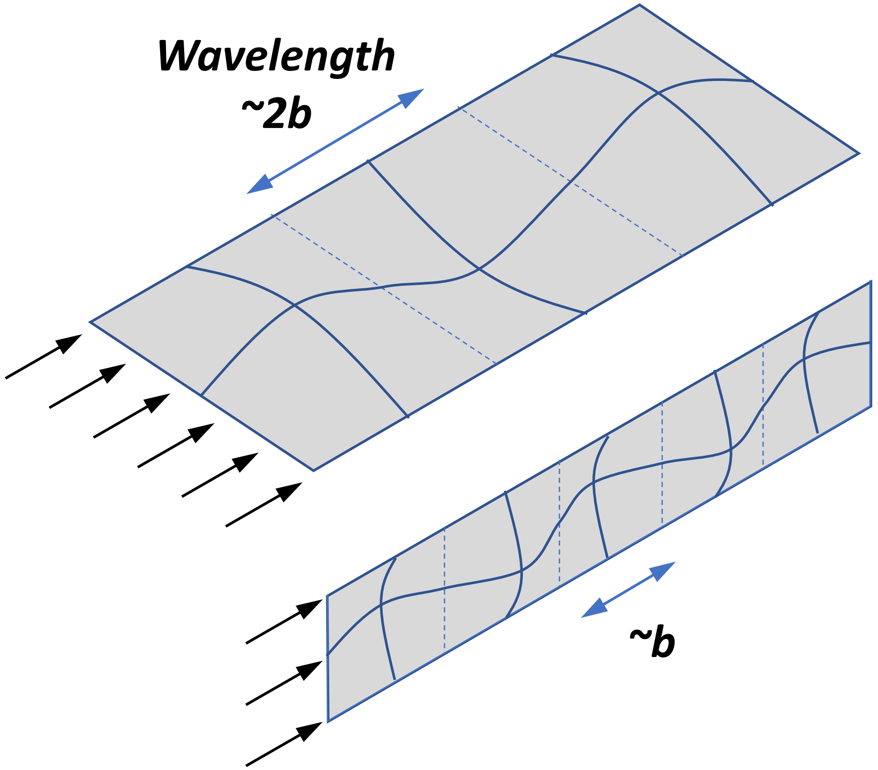
Note: the plasticity correction factor η is not the same for plates & columns

Thin-Walled Section Composed of Flat Plates



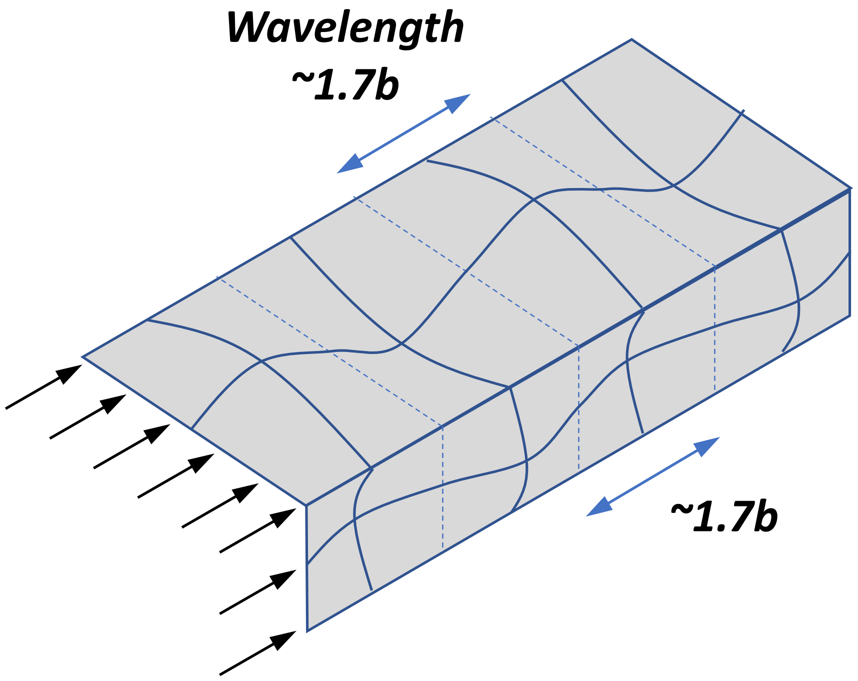
2 x 1 Hollow Rectangular Tube

Buckling as separate plates



Independent wave lengths ~square for each plate

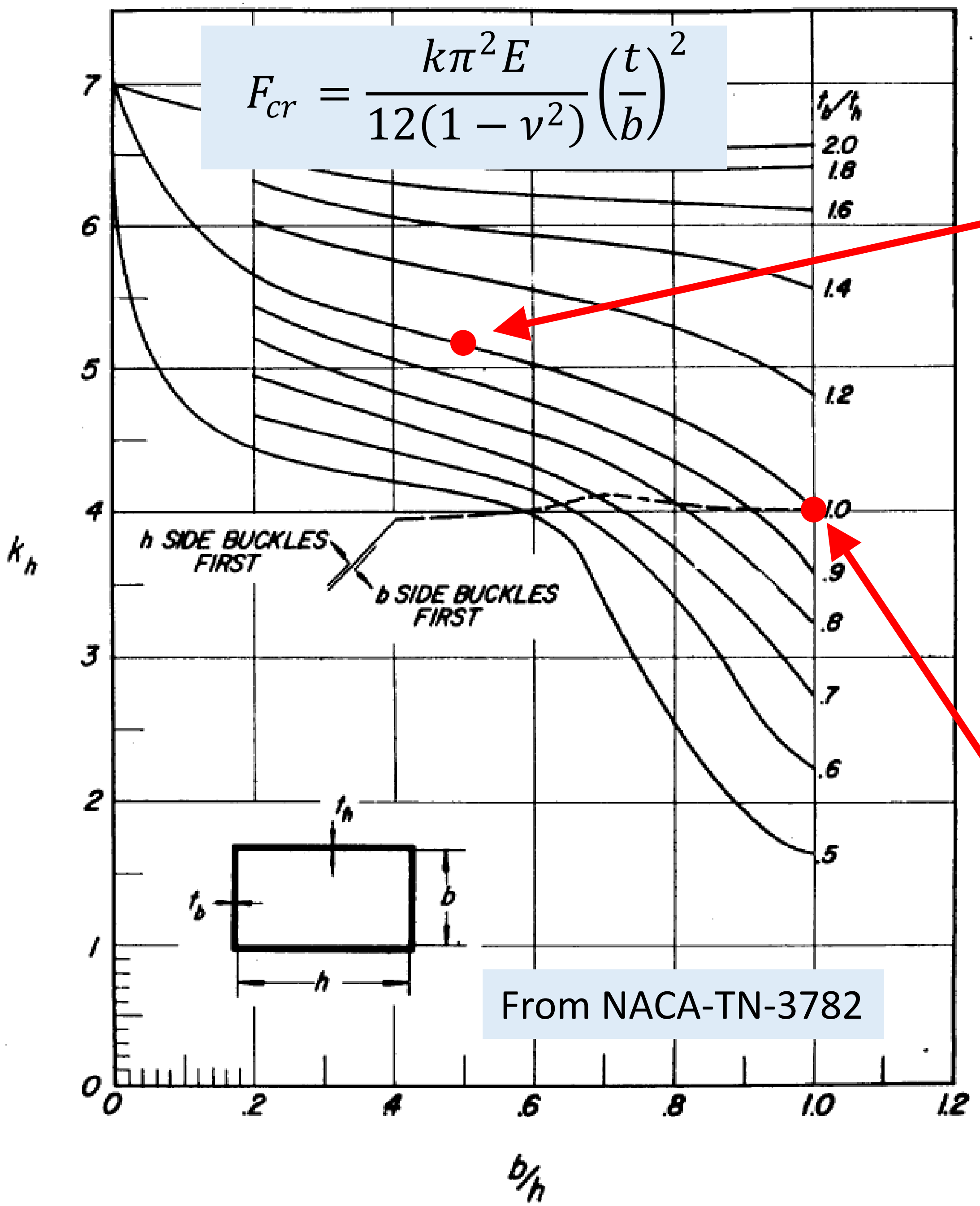
Buckling as coupled plates



Compatible wave length is >b but <2b

Note: enforcement of compatible wave length increases buckling load

$$F_{cr} = \frac{k\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2$$



From NACA-TN-3782

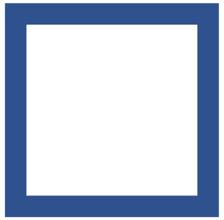
2 x 1 uniform thickness tube

k = 5.15

(29% > simply-supported)

Short side supports long side – buckling coefficient k greater than simply-supported

square uniform thickness tube

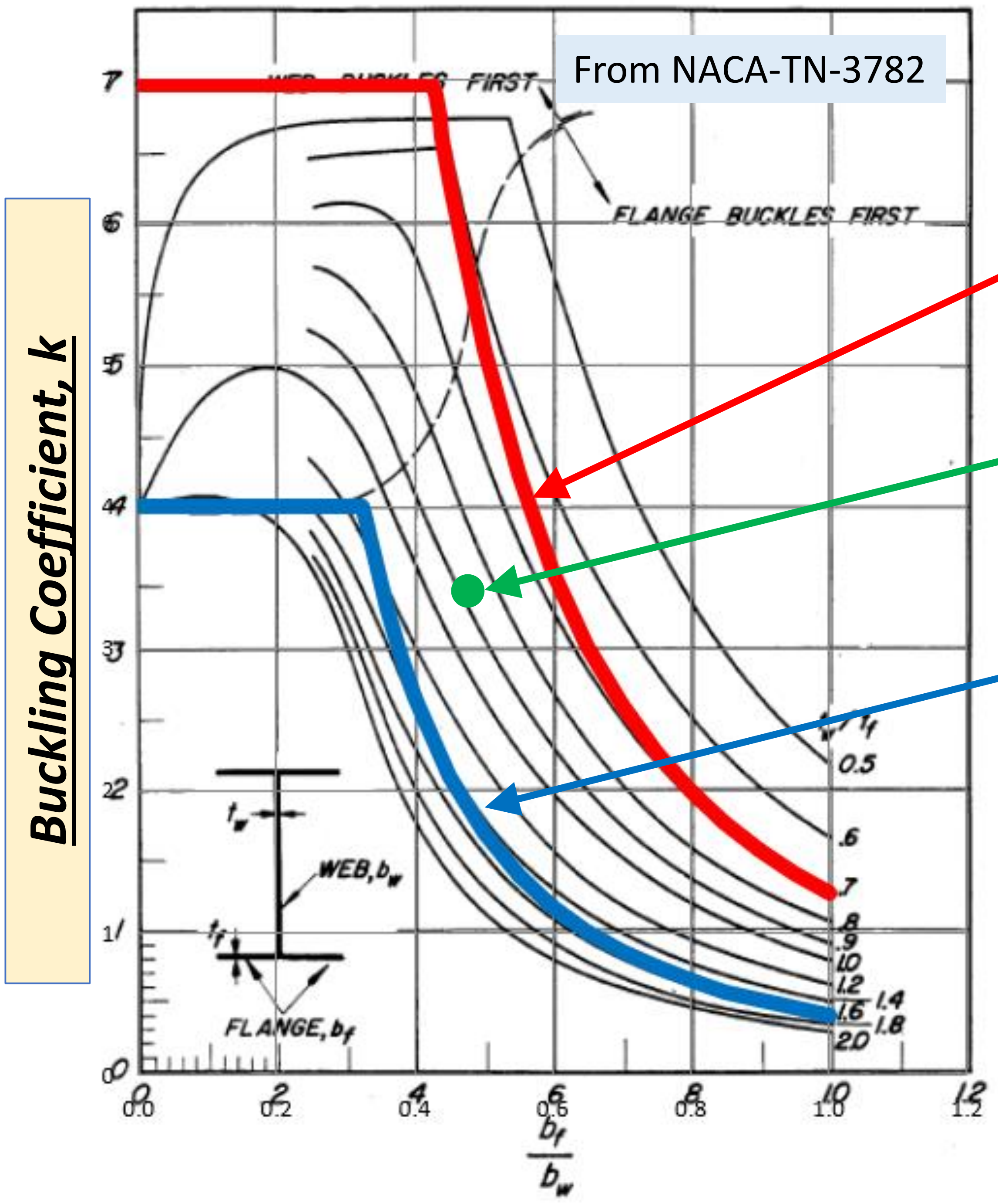


k = 4 (same as simply-supported)

Neither side can support the other – buckling coefficient k same as simply-supported

Another Example:

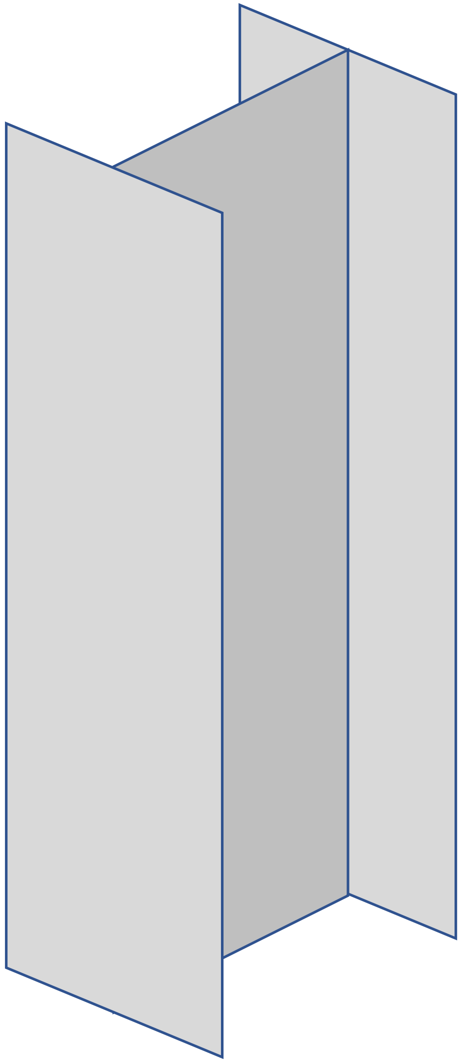
Local Buckling of I-Section



RED curve if assume fixed edges

Actual buckling coefficient depends on relative b/t of adjacent segments

BLUE curve if assume simply-supported edges

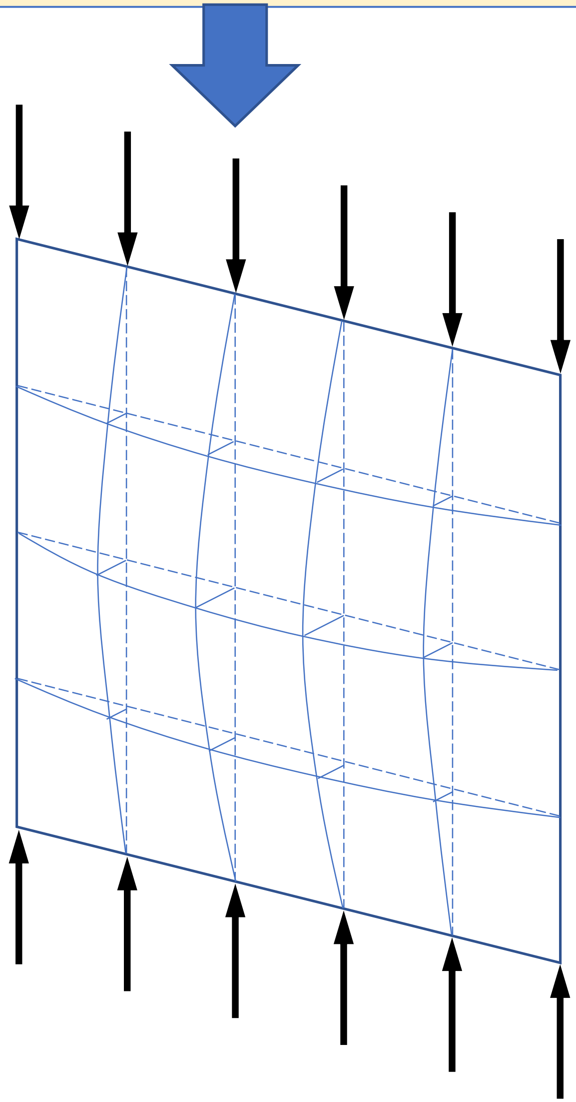


Post-buckling

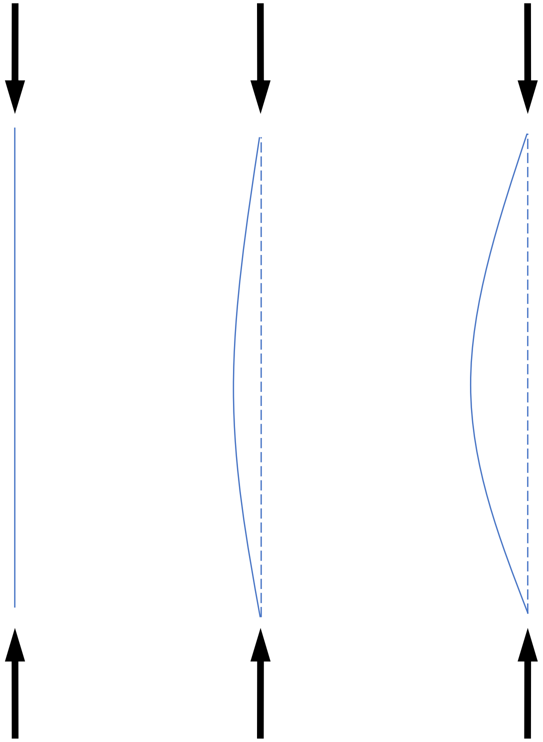
About Post-Buckling

- Buckling is not necessarily failure
  - The design criteria on a specific project may dictate no buckling at some specified load (e.g. no buckling at limit, no buckling at ultimate), but by failure here is meant the maximum load the component can carry
- Thin plates and sections composed of thin plates can often carry significant load beyond buckling (i.e. they have post-buckling strength)
- Two main things happen in the post-buckling range:
  - The overall stiffness of the part is reduced from the pre-buckling stiffness
  - The internal loads/stresses redistribute compared to the pre-buckling distribution
- Post-buckling analysis is inherently nonlinear due to both large displacements and plasticity, hence classical hand analysis methods rely on semi-empirical equations correlated with lots of test data

Buckled shape of a simply-supported square plate under axial compression



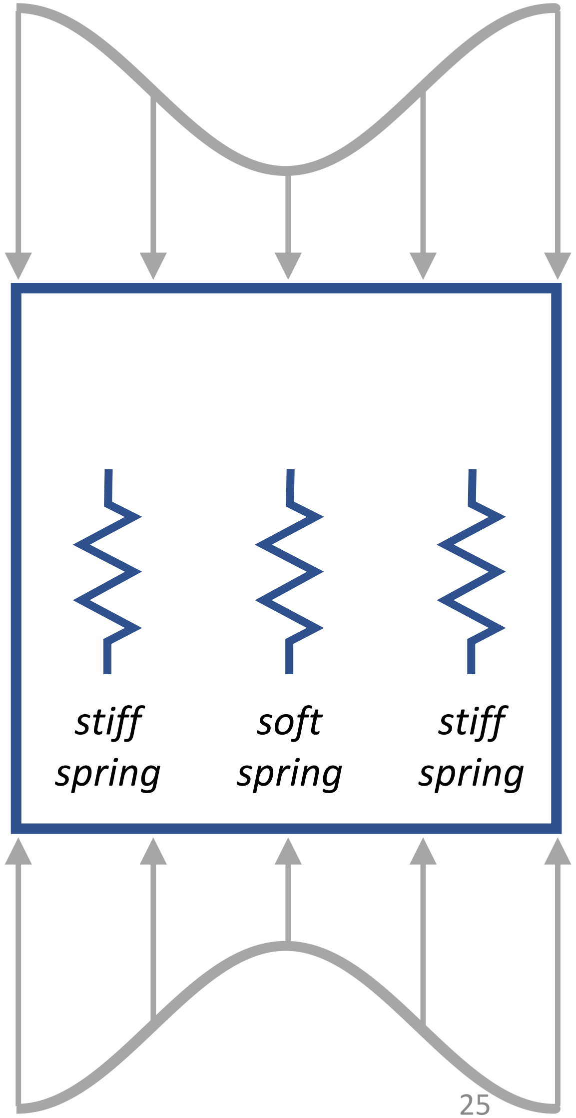
Deflection of Vertical Strips



Near edge:
High stiffness

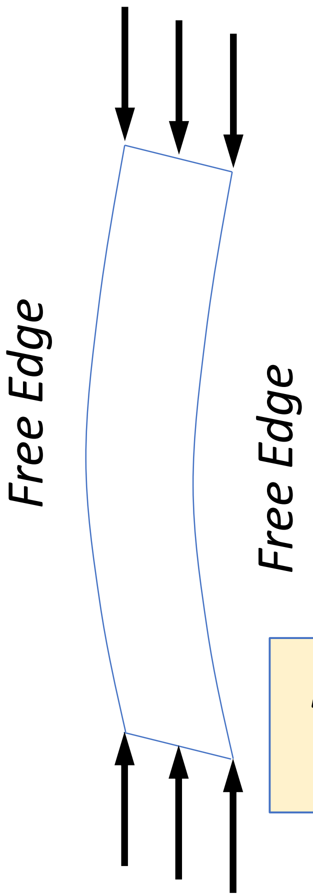
Near middle:
Low stiffness

<u>Parallel springs</u>
Stiff springs carry more load → leads to nonlinear distribution



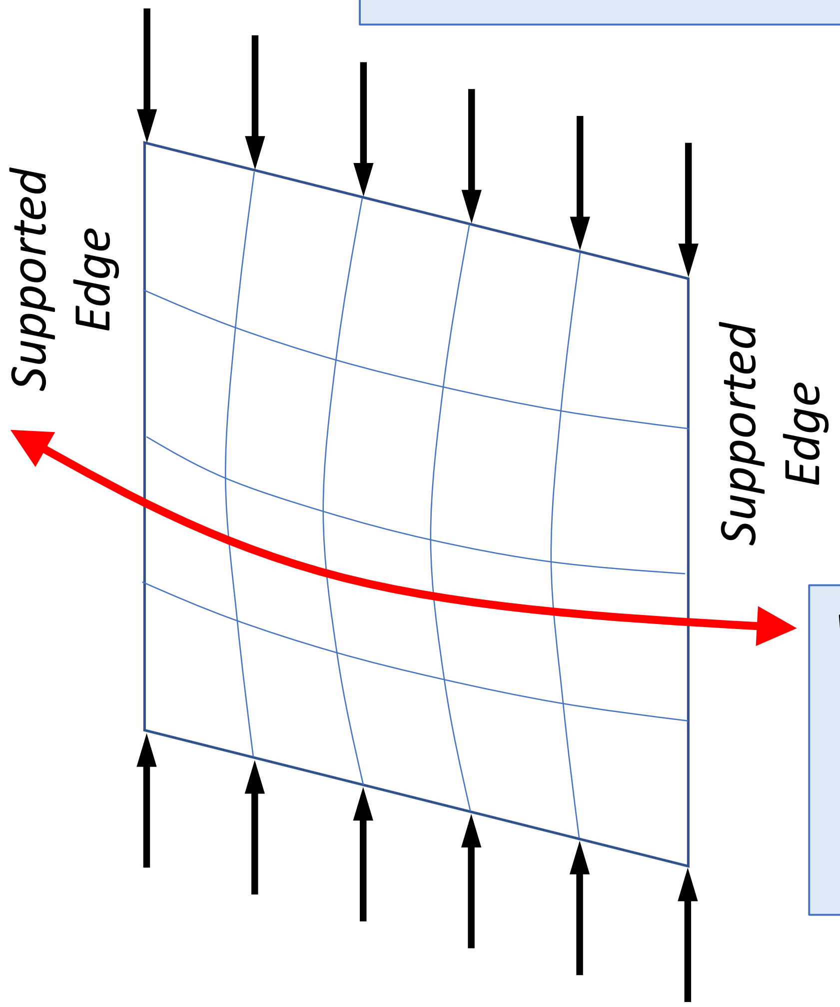
Buckling Shapes Compared

Column Buckling



Essentially no post-buckling strength

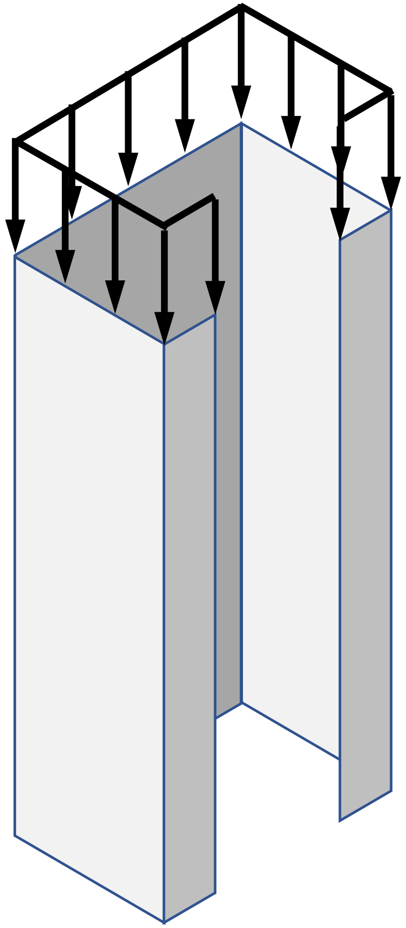
Plate Buckling



When vertical strips deflect, lateral strips provide restraint – prevents immediate collapse

Redistribution of Internal Loads/Stresses

Pre-Buckling



Walls flat, compression load uniformly distributed

Post-Buckling

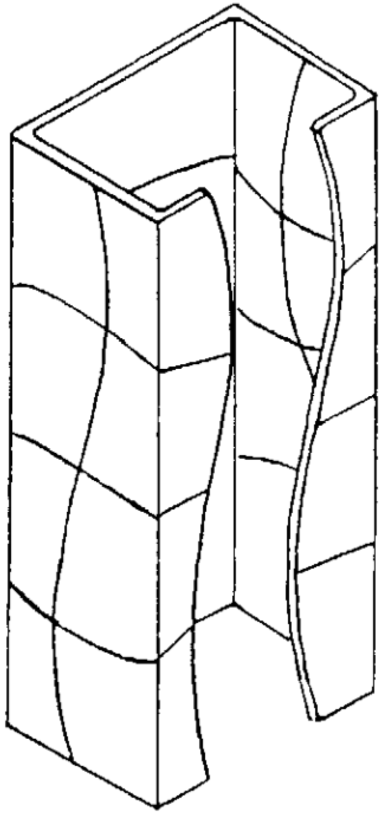


Fig. 5.3.1 Cross-sectional distortion.

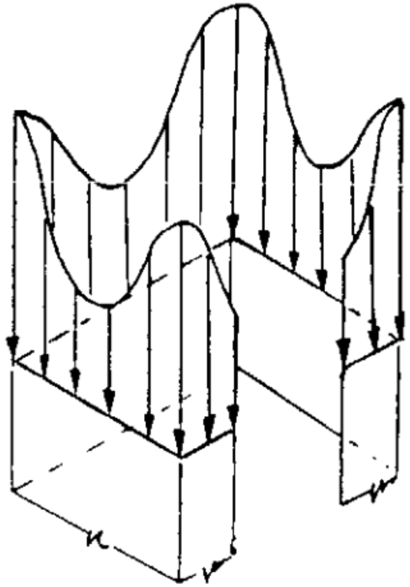


Fig. 5.3.2 Stress distribution.

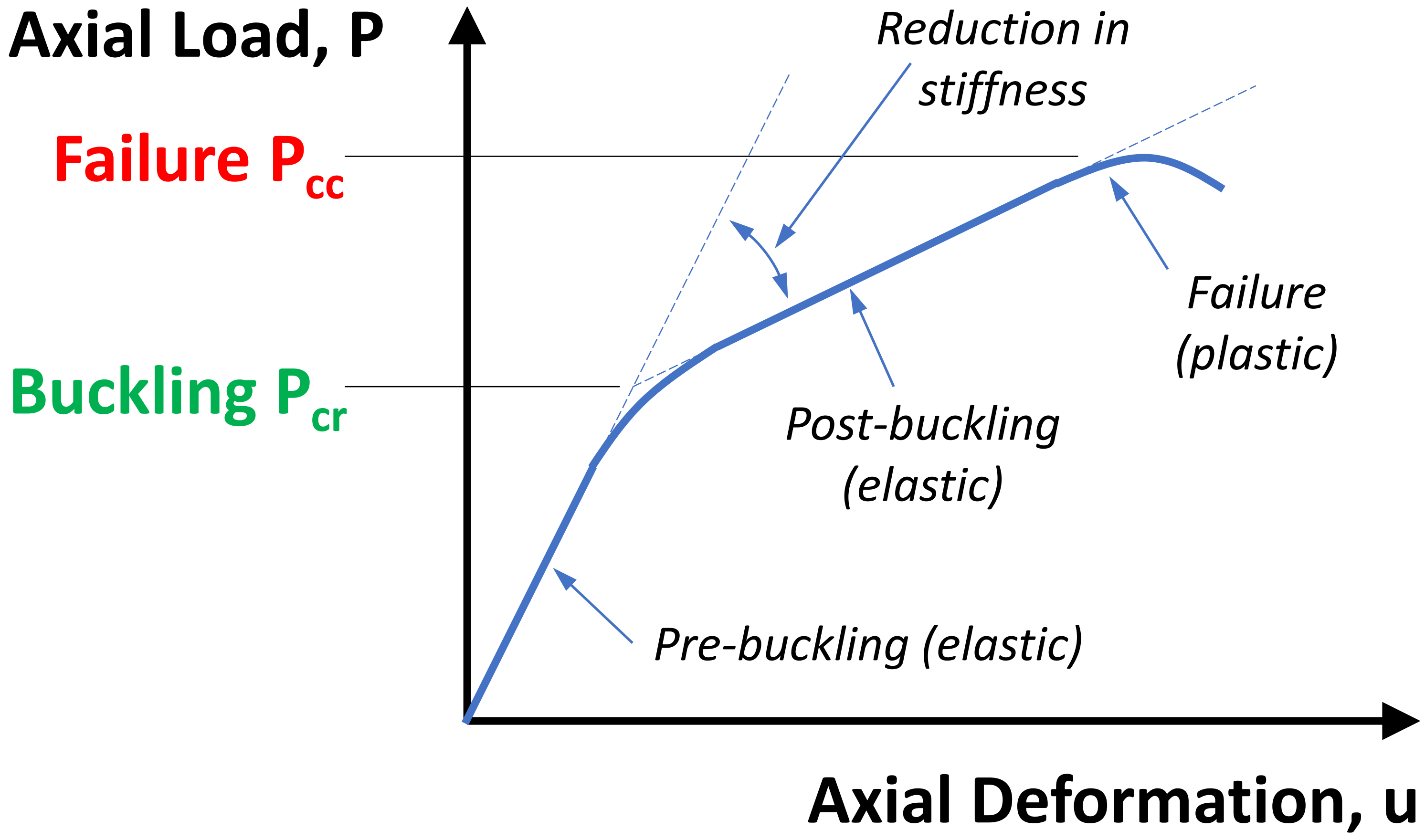
Waves in walls, compression load peaks near corners

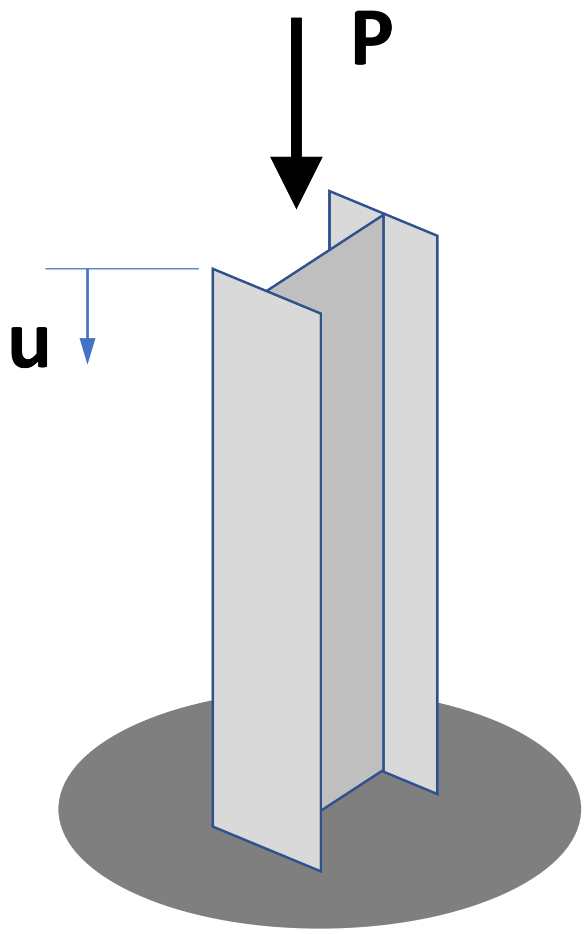
From: NASA Astronautic Structures Manual

Crippling Failure

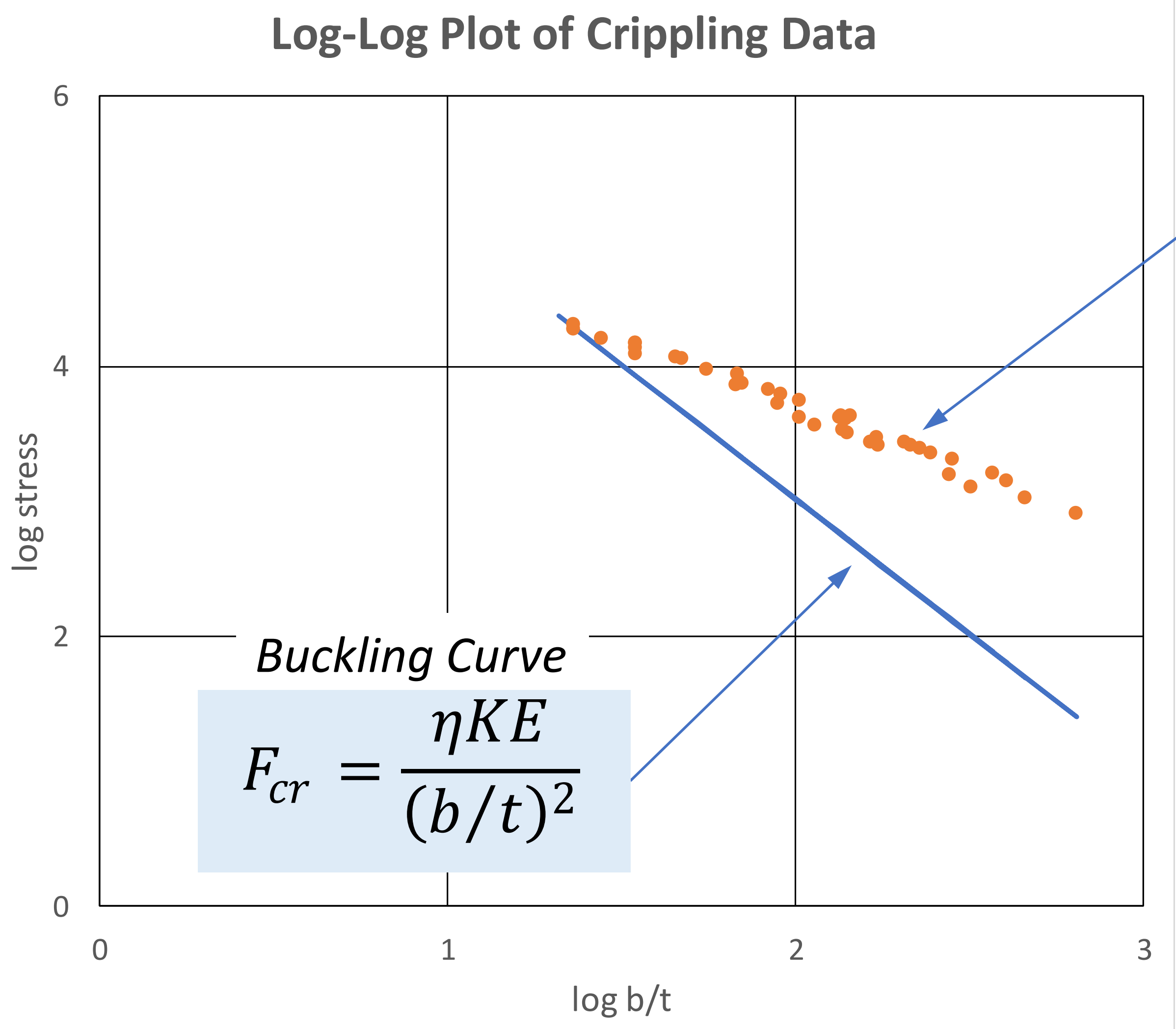
Load-Deflection Behavior of a Thin-Walled Column

P_{cr} = buckling load
P_{cc} = failure (crippling) load





Semi-Empirical Crippling Equations



Buckling Curve

$$F_{cr} = \frac{\eta K E}{(b/t)^2}$$

<u>Test Failure Stress Data</u>

Straight line on a log-log plot indicates power law is a reasonable model

NACA-TN-3784 Power-Law Relation:

$$F_{cc} = \alpha\left(F_{cy}\right)^{n}\left(F_{cr}\right)^{1-n}$$

For n=1 → $F_{cc} = \alpha F_{cy}$

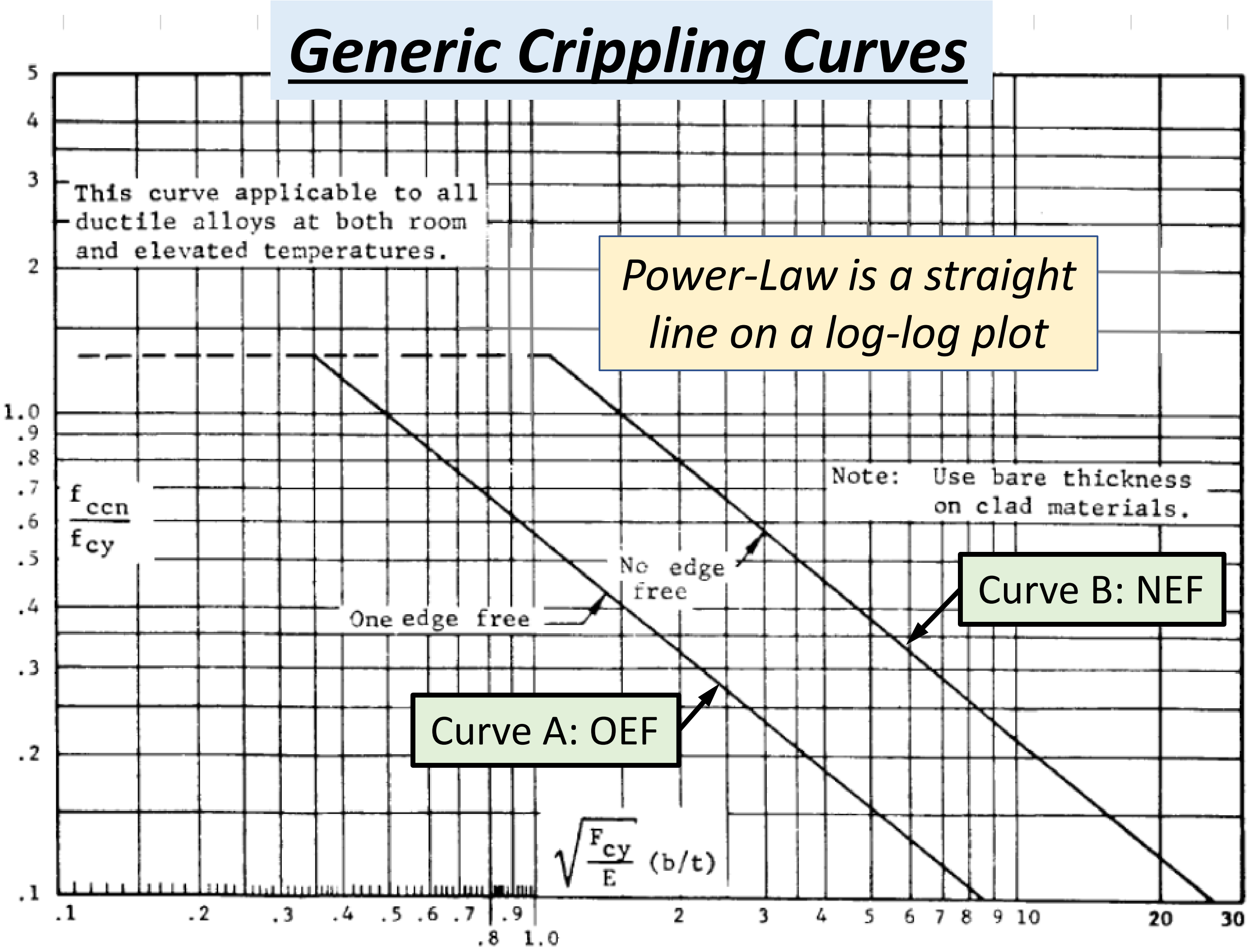
For n=0 → $F_{cc} = \alpha F_{cr}$

For 0<n<1 → $\alpha F_{cr} \le F_{cc} \le \alpha F_{cy}$

Crippling Stress is somewhere between Buckling Stress and Yield Stress

I say semi-empirical because the basic form of the equations is based on theory, then the parameters of those equations are "tuned" to match test data

Typical Crippling Curves (many variations exist)



From: NASA Astronautic Structures Manual

Curve A: OEF

$$\frac{F_{cc}}{F_{cy}} = 0.56\left[\sqrt{\frac{F_{cy}}{E}}\left(\frac{b}{t}\right)\right]^{-0.8}$$

Can rewrite as → $F_{cc} = 0.56\,\frac{F_{cy}^{0.6}E^{0.4}}{(b/t)^{0.8}}$

Curve B: NEF

$$\frac{F_{cc}}{F_{cy}} = 1.4\left[\sqrt{\frac{F_{cy}}{E}}\left(\frac{b}{t}\right)\right]^{-0.8}$$

Can rewrite as → $F_{cc} = 1.4\,\frac{F_{cy}^{0.6}E^{0.4}}{(b/t)^{0.8}}$

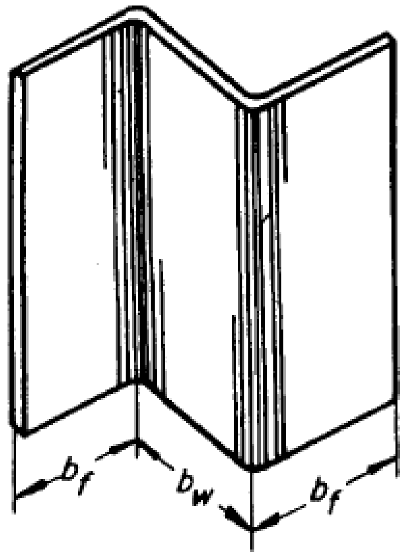
Buckling & Crippling Equations Compared

	No Edge Free (NEF)	One Edge Free (OEF)
Buckling *Note: buckling eqn is power law with exponent = -2*	$F_{cr} = \frac{\eta k \pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2$ *where k = 4 for S.S.*	$F_{cr} = \frac{\eta k \pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2$ *where k = 0.43 for S.S.*
Crippling *Note: this crippling eqn is power law with exponent = -0.8*	$F_{cc} = 1.4\frac{F_{cy}^{0.6}E^{0.4}}{(b/t)^{0.8}}$	$F_{cc} = 0.56\frac{F_{cy}^{0.6}E^{0.4}}{(b/t)^{0.8}}$

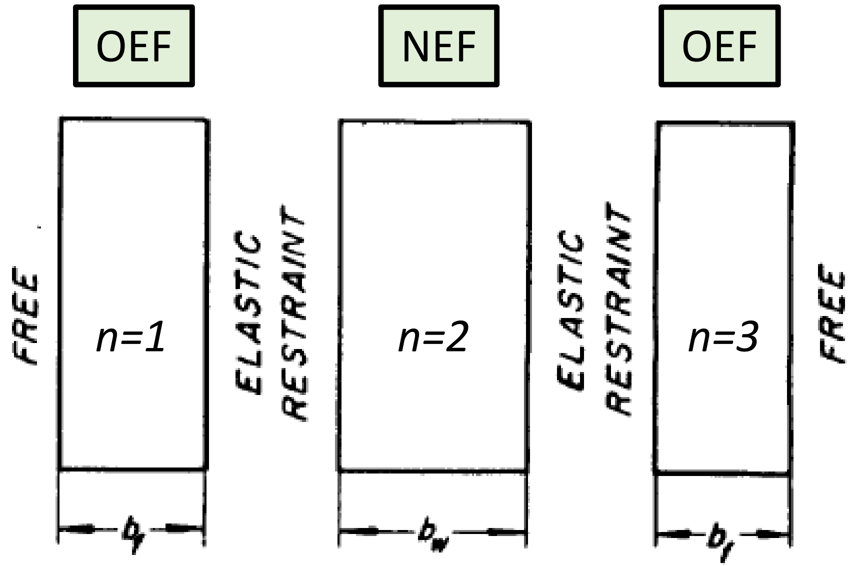
These crippling equations are representative, actual equations depend on a variety of factors including: alloy, product form (extrusion, formed sheet, etc.) as well as airplane company

Crippling Stress of a Thin-Walled Section

Typical Thin-Walled Section



Break into "n" segments

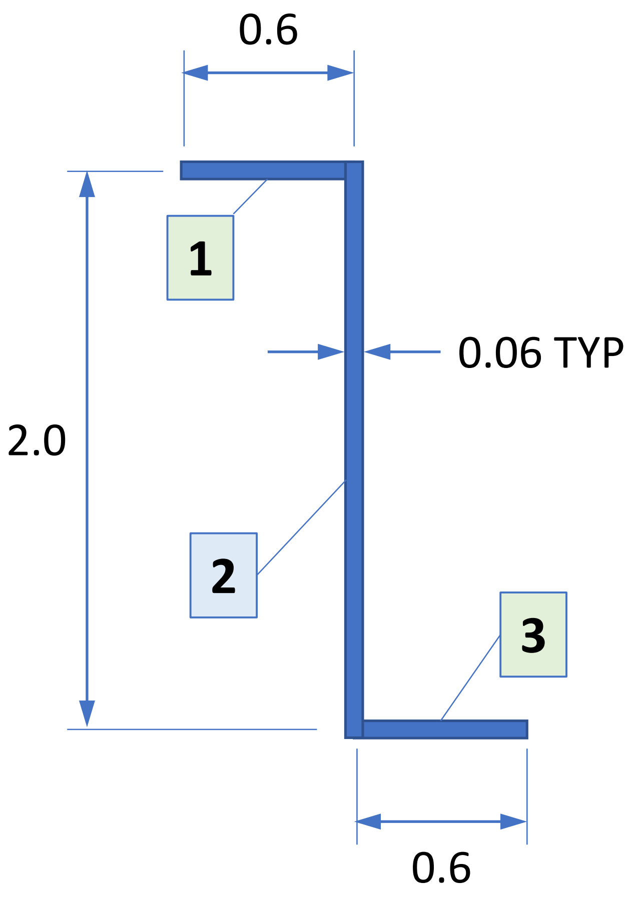


Crippling stress of section is area weighted average of crippling stresses of segments

$$F_{cc} = \frac{\sum F_{ccn} A_n}{\sum A_n}$$

F_{ccn} = crippling stress of n^{th} segment (use either NEF or OEF equation)
A_n = area of n^{th} segment

Example Crippling Stress Calculation

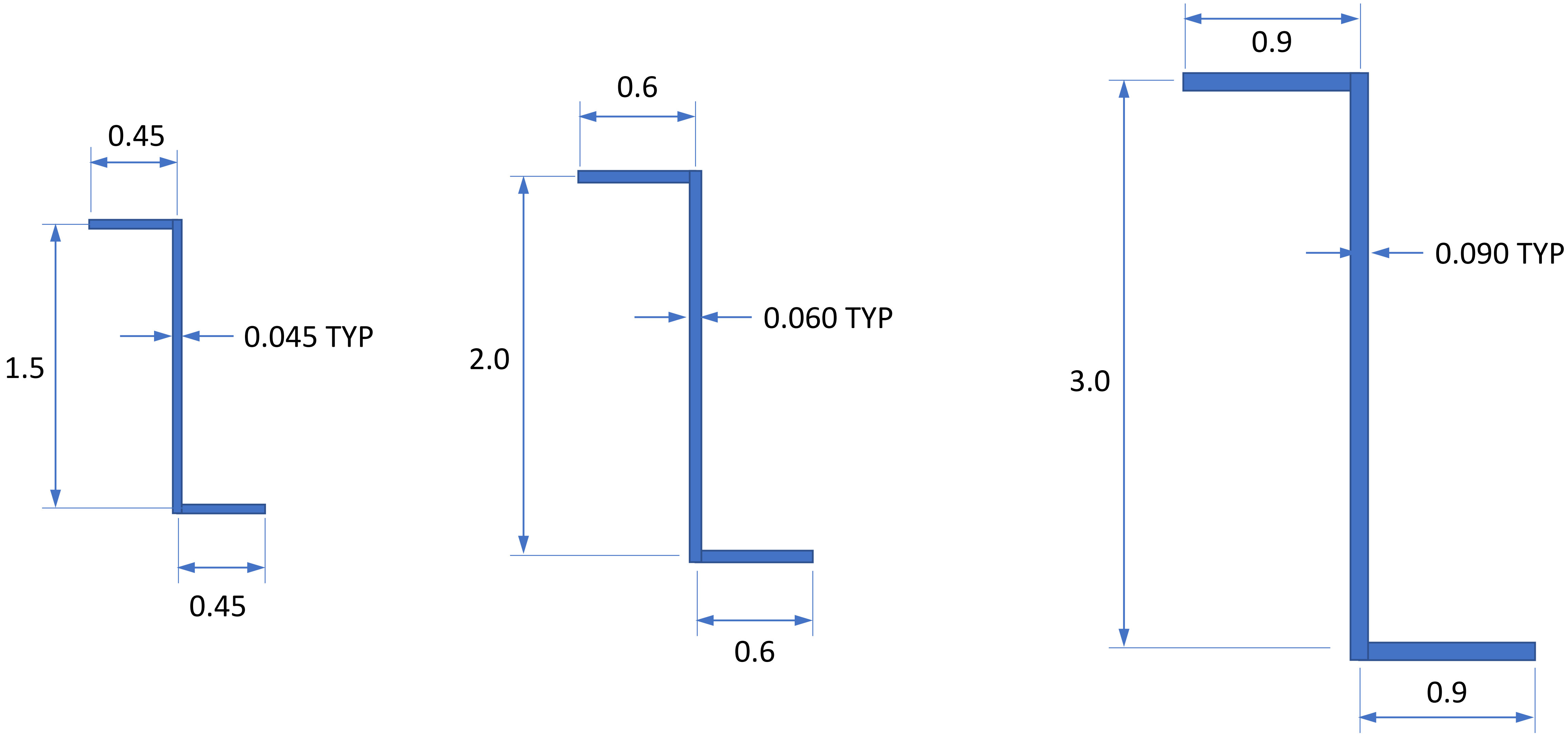


	Aluminum				Crippling Stress:		
	E	1.00E+07	psi		Fcc	31391	psi
	Fcy	40000	psi				
	Segment	b	t	b/t	A	Fcc	Pcc
		in	in		in^2	psi	lb
OEF	1	0.60	0.060	10.00	0.036	32316	1163
NEF	2	2.00	0.060	33.33	0.120	30836	3700
OEF	3	0.60	0.060	10.00	0.036	32316	1163
				sums -->	0.192		6027

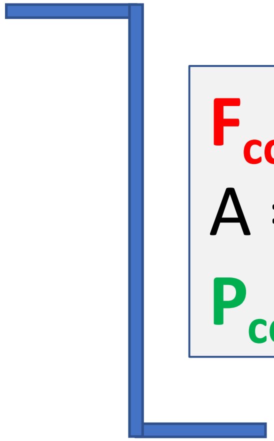
For NEF segments use: $$F_{cc} = 1.4\frac{F_{cy}^{0.6}E^{0.4}}{(b/t)^{0.8}}$$

For OEF segments use: $$F_{cc} = 0.56\frac{F_{cy}^{0.6}E^{0.4}}{(b/t)^{0.8}}$$

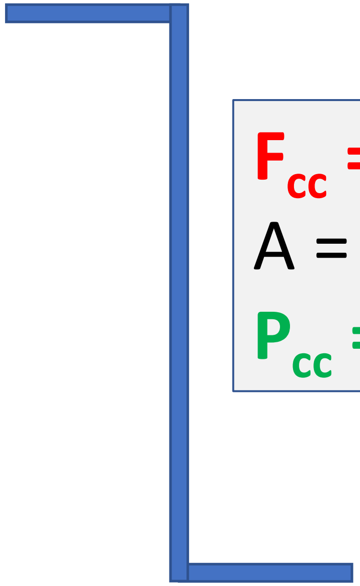
Question: which has the highest crippling stress?



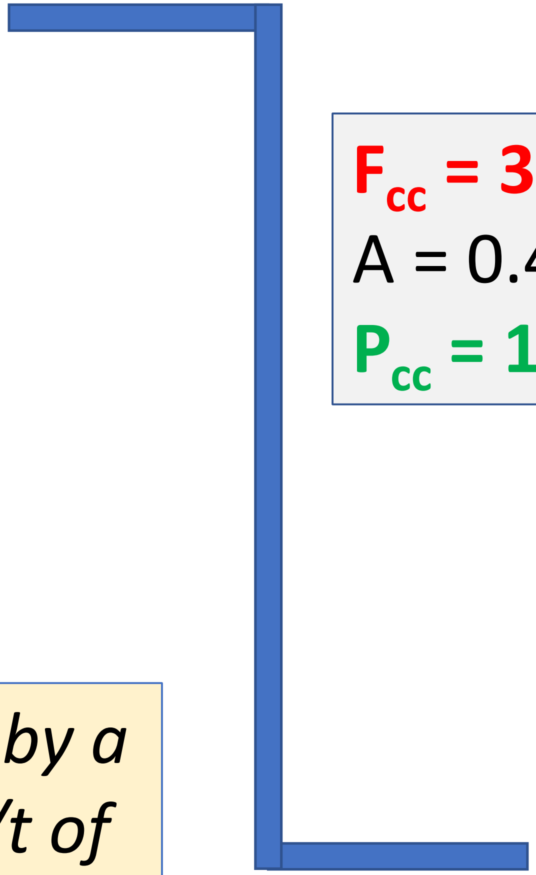
Answer: they all have the same crippling stress



F_{cc} = 31391 psi
$A = 0.108\ in^2$
P_{cc} = 3390 lb



F_{cc} = 31391 psi
$A = 0.192\ in^2$
P_{cc} = 6027 lb

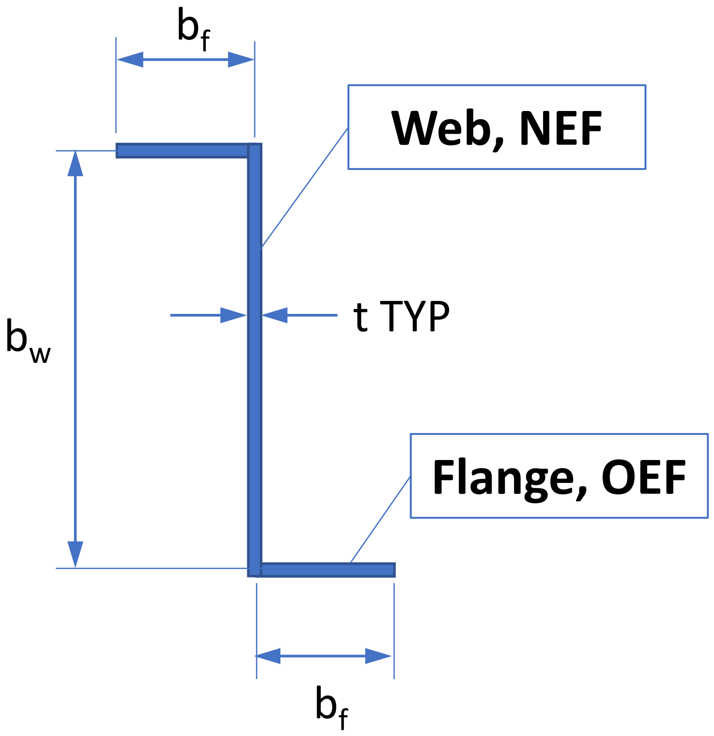


F_{cc} = 31391 psi
$A = 0.432\ in^2$
P_{cc} = 13561 lb

Geometrically similar shapes (all dimensions scaled by a common factor) have same crippling stress since b/t of the segments remains the same. But since the area scales, the total crippling LOAD P_{cc} does change.

Section with “Balanced” Crippling

For a formed Z-section (uniform thickness), determine the ratio of flange width to web height such that the flanges and the web have the same crippling stress



$$F_{cc,NEF} = F_{cc,OEF}$$

$$1.4\frac{F_{cy}^{0.6}E^{0.4}}{(b_w/t)^{0.8}} = 0.56\frac{F_{cy}^{0.6}E^{0.4}}{\left(b_f/t\right)^{0.8}}$$

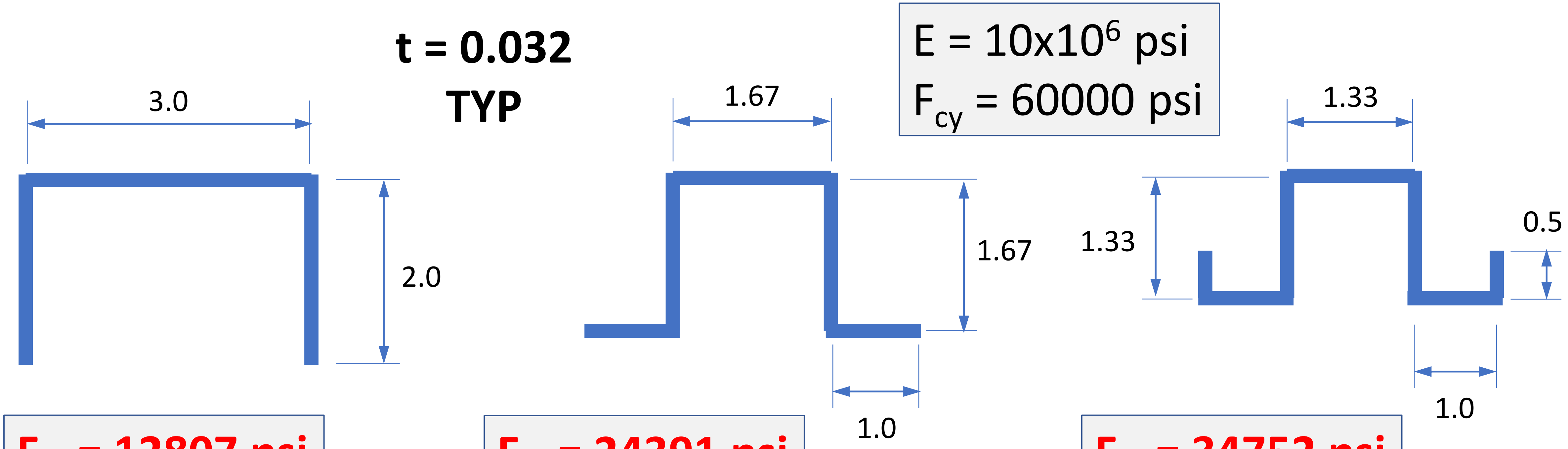
$$\left(\frac{b_f}{b_w}\right)^{0.8} = \frac{0.56}{1.4}$$

$$\frac{b_f}{b_w} = 0.32$$

→ The flange width should be about 1/3 the web height for web & flanges to cripple at the same stress

More Examples:

Keep b/t down when high crippling loads are needed



F_{cc} = 12807 psi
A = 0.224 in²
P_{cc} = 2859 lb

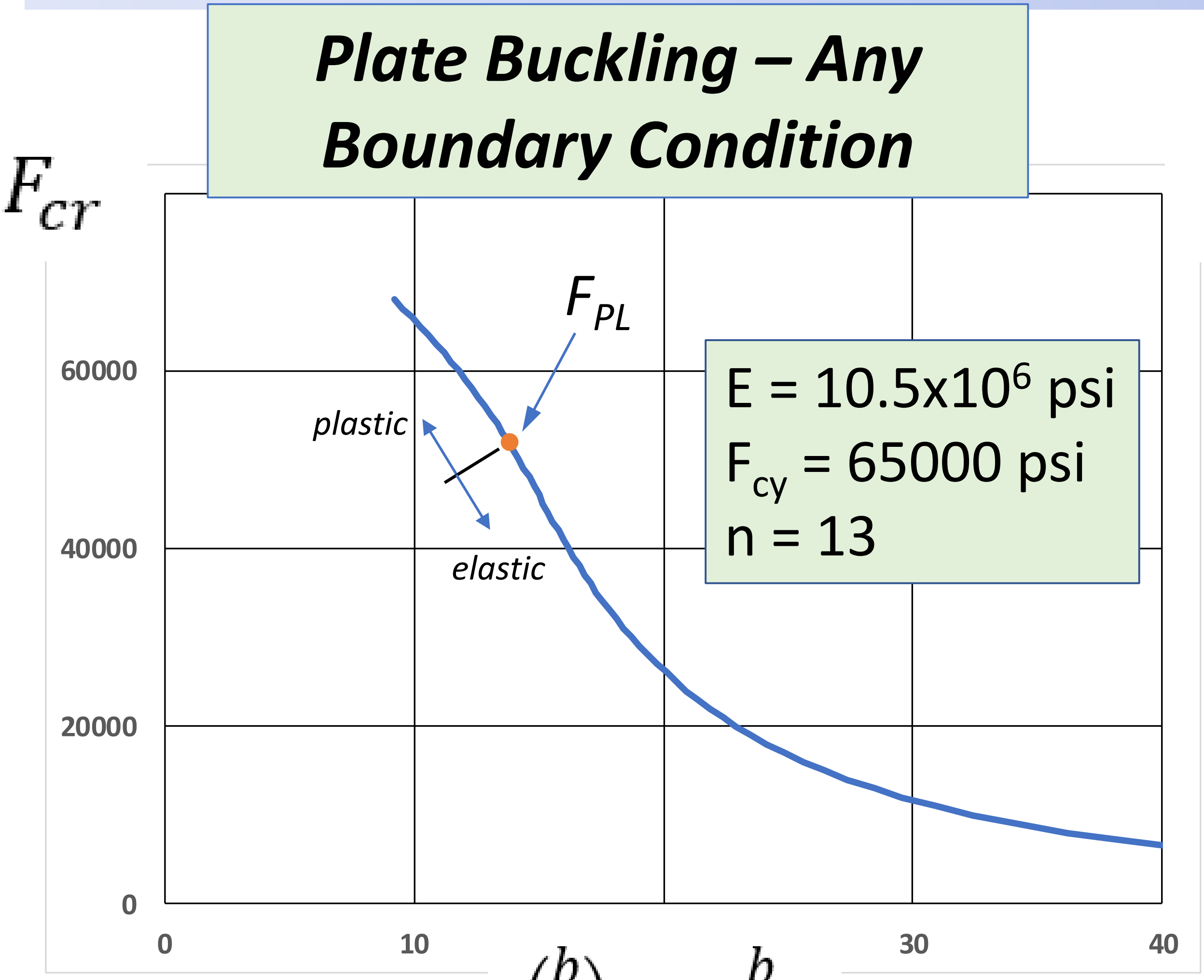
F_{cc} = 24391 psi
A = 0.224 in²
P_{cc} = 5463 lb

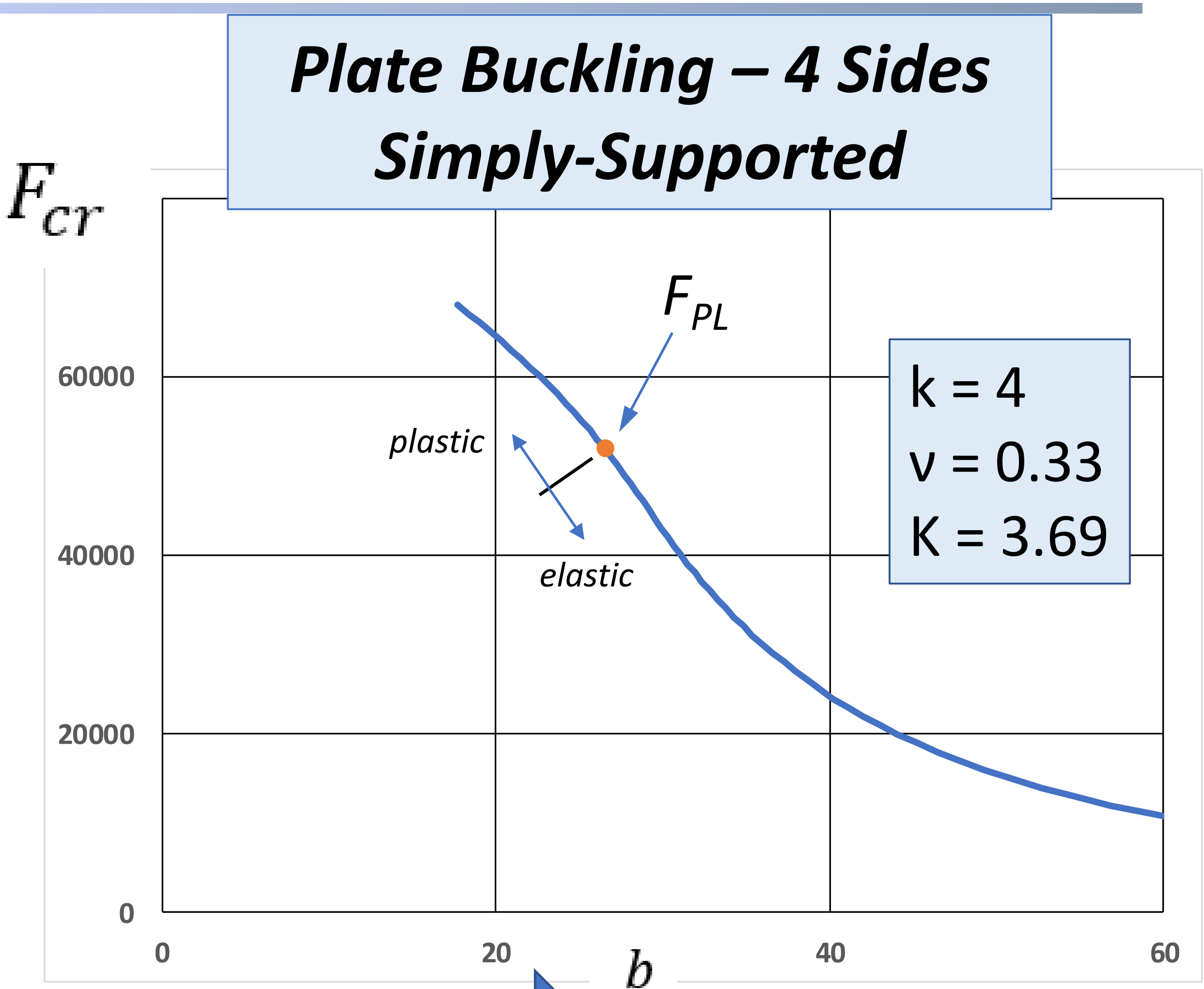
F_{cc} = 34752 psi
A = 0.224 in²
P_{cc} = 7785 lb

Can get higher crippling load for same weight by using smaller b/t segments

Let's Re-Plot the Buckling and Crippling Curves so we can Overlay Them

Re-Plot Plate Buckling Curve for 4SSS

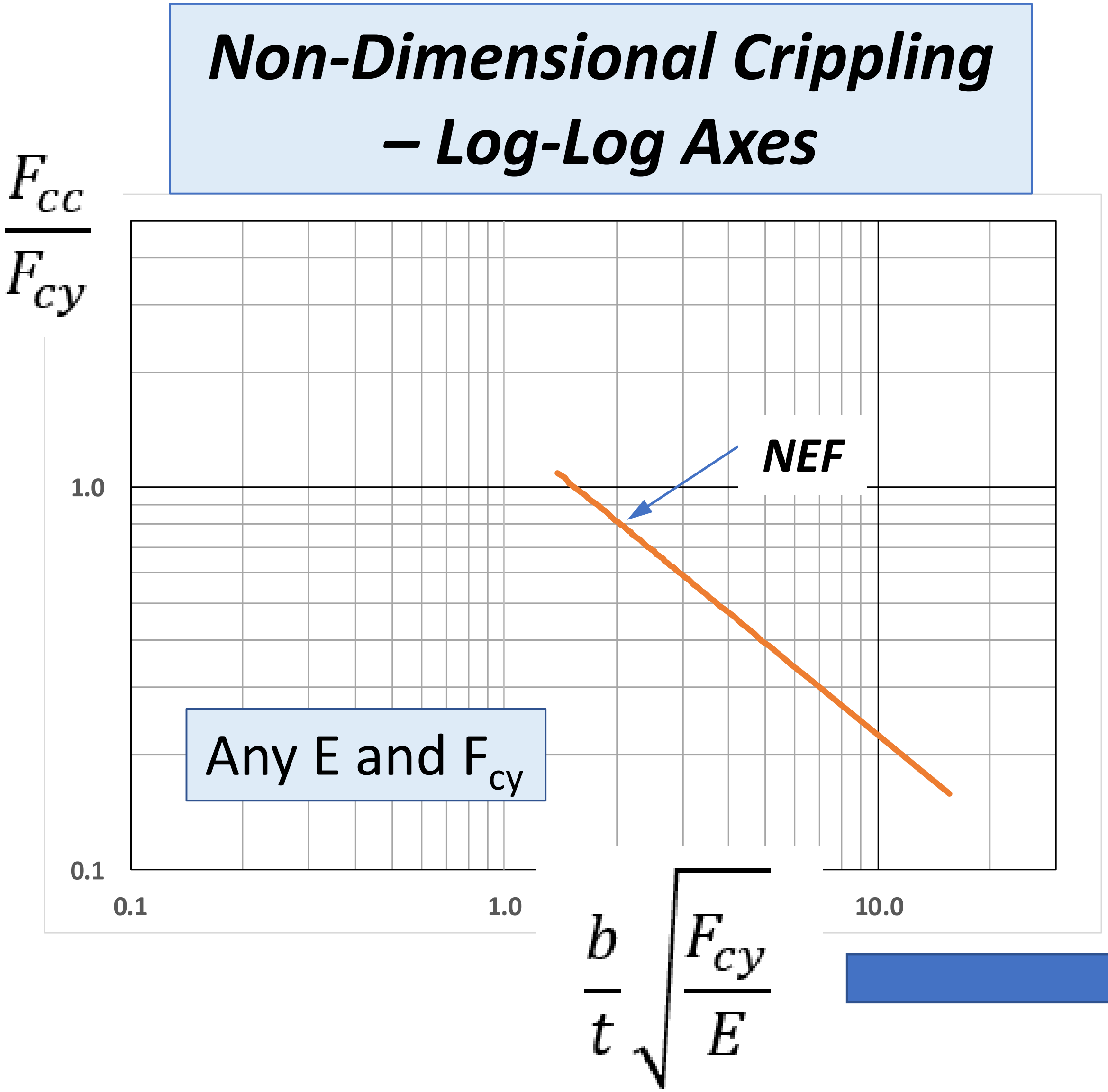


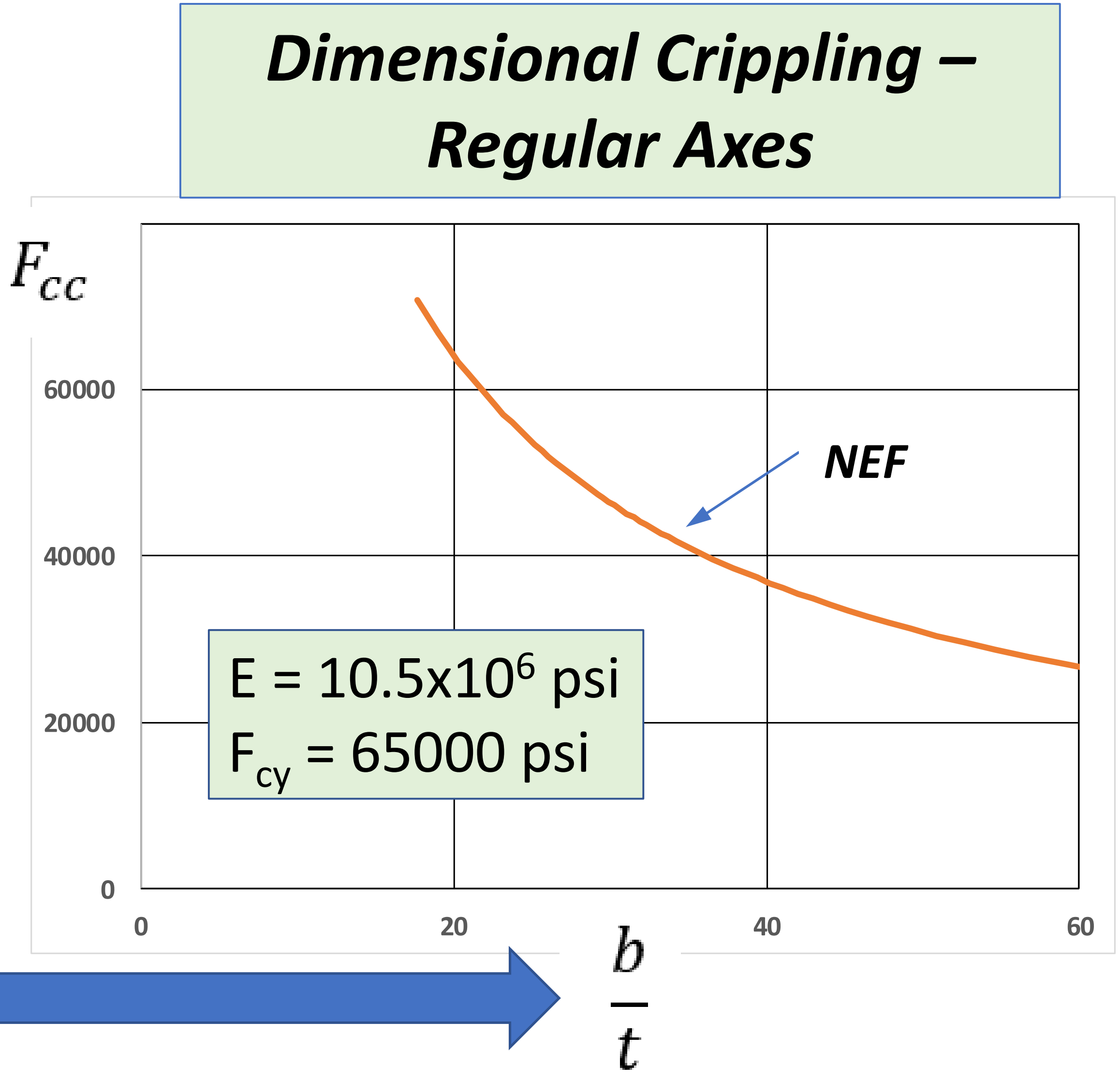


B.C. embedded in $(b/t)_e$

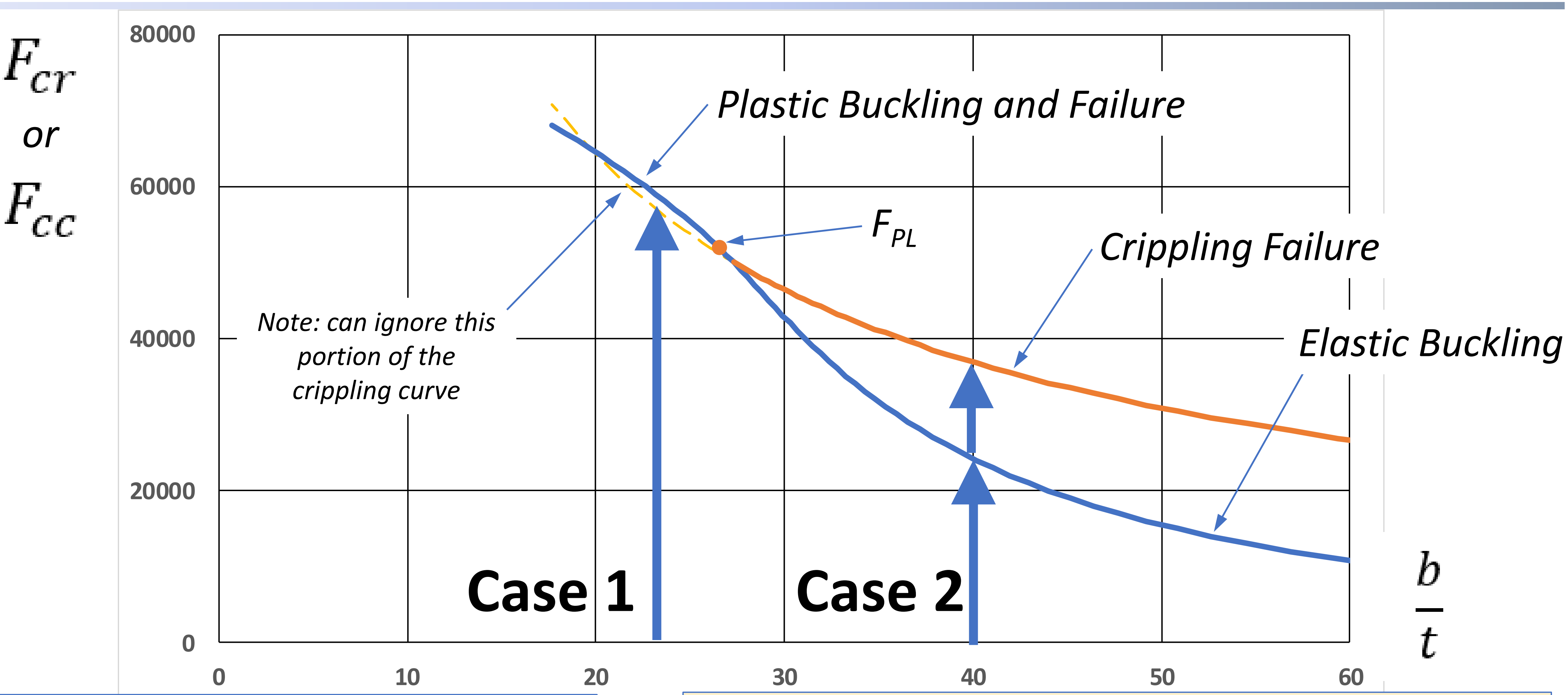
$$\left(\frac{b}{t}\right)_e = \frac{b}{t\sqrt{K}} \rightarrow \frac{b}{t}$$

Re-Plot Crippling Curve as F_{cc} vs b/t





Now Can Overlay Buckling and Crippling Curves

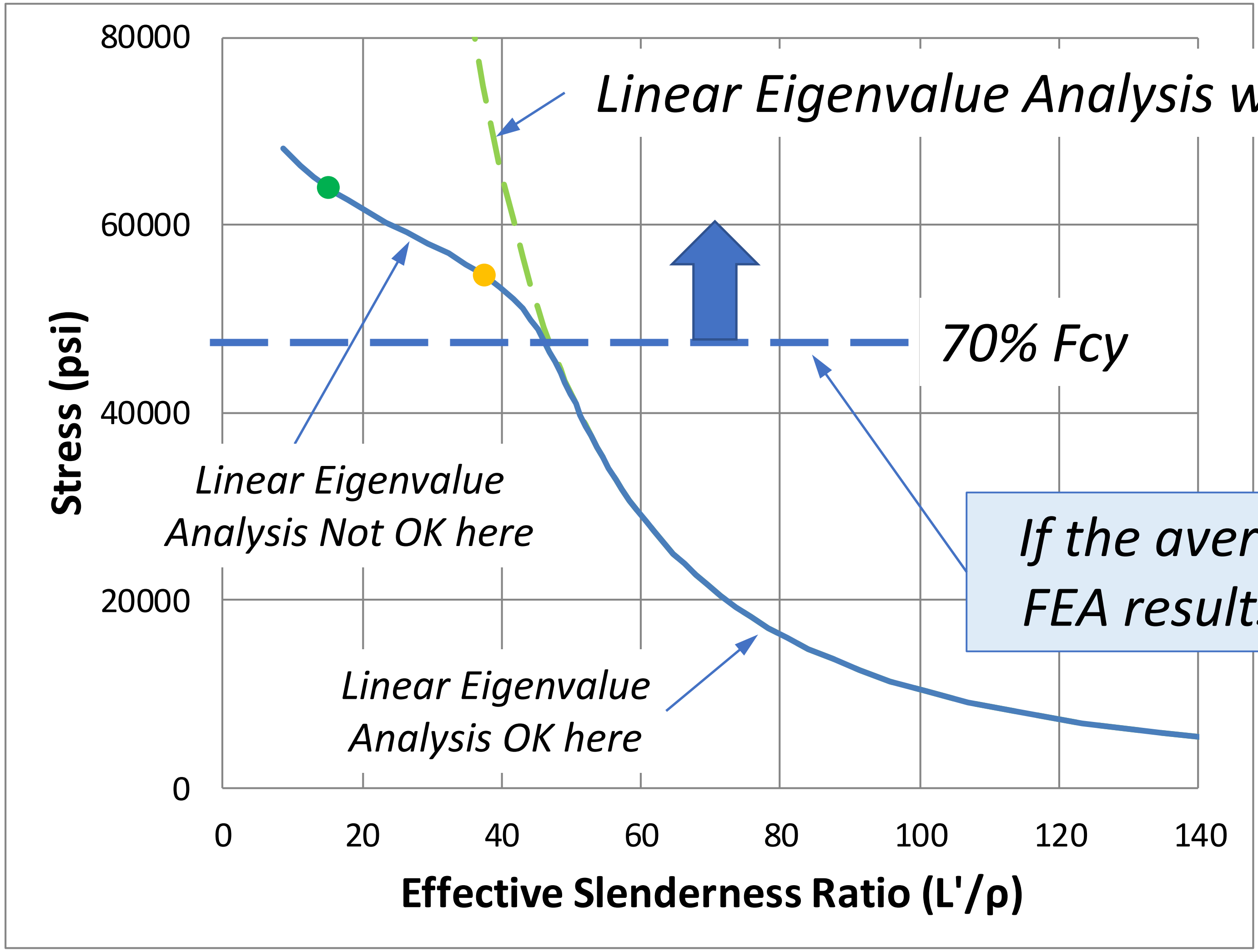


Case 1: *b/t=22, plastic buckling & immediate failure occurs at ~60 ksi*

Case 2: *b/t=40, elastic buckling occurs at ~24 ksi followed by crippling failure at ~37 ksi*

Final Thoughts

Comment on FEA Linear Eigenvalue Analysis

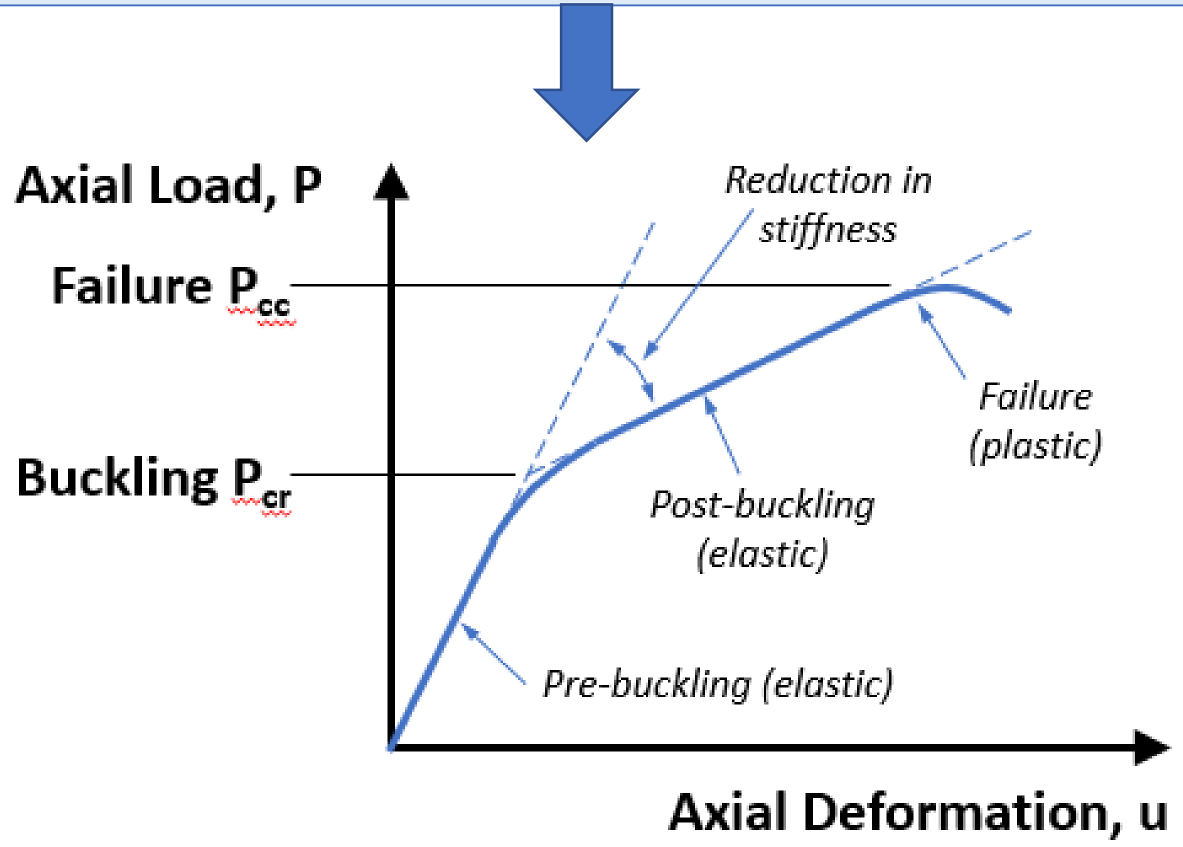


Linear Eigenvalue Analysis does not include a Plasticity Correction Factor!

If the average stresses are > about 70% Fcy, the FEA results will <u>over-estimate</u> the buckling load

Comment on Post-Buckling using Nonlinear FEA

Can use Nonlinear FEA to track the load-deflection behavior as shown here



Large displacements required

Nonlinear material required for failure, can do limited post-buckling analysis with linear material

Classical hand analysis methods have been correlated with lots of test data over many years

Due to many real-world uncertainties, Nonlinear FEA results should be correlated to test as well

Other important buckling related topics not discussed here: shear buckling, diagonal tension, buckling of stiffened panels, etc.

Summary

- Plate buckling is governed by the plate width *b*. This is important to know when laying out structure because it affects spar spacing, stiffener spacing, etc.
- For plates and thin-walled sections composed of plates, buckling is not necessarily failure
  - Very thin sections buckle elastically, then fail by crippling at higher loads
  - Relatively thick sections buckle plastically, and fail at about the same load
  - Project design criteria may not allow buckling below a certain load. Not meeting that criteria may be considered a type of “failure”, but is not failure as meant here.
- Crippling and buckling are not the same
  - Crippling is defined as the maximum load a thin-walled section can carry, and by definition, it happens after initial buckling
- Buckling & Post-Buckling analyses can be performed using Finite Element Analysis, but there are important items to consider before trusting results

References: NACA Handbook of Structural Stability

"Composite" → assembly of flat plates
Failure, strength → max load, crippling

1. NACA-TN-3781, Buckling of Flat Plates
2. NACA-TN-3782, Buckling of Composite Elements
3. NACA-TN-3783, Buckling of Curved Plates and Shells
4. NACA-TN-3784, Failure of Plates and Composite Elements ← Highly recommended for crippling equation derivations
5. NACA-TN-3785, Compressive Strength of Flat Stiffened Panels
6. NACA-TN-3786, Strength of Stiffened Curved Plates and Shells

Many of the figures in Bruhn came directly from these reports

Learn something new every day…